# Randolph High School

## Mathematics Department

**To:** Prospective Calculus A Students

**From:** The Calculus A Teachers

**Subject:** Summer Assignment

Calculus A is a rigorous yearlong course that will prepare you for additional higher level math courses at the college you will be attending. It covers an extensive selection of mathematical topics and is a very demanding course.

A requirement for success in the Calculus A course is that you have absorbed and are able to apply many concepts learned in your previous math courses. In an attempt to ensure that you are sufficiently prepared for Calculus, you are asked to complete the attached "Summer Review Packet." This attachment contains a summary of many topics from your previous math courses. Please read/study this review material and do all problems. All problems should be done in order, clearly labeled, neatly written, and **showing all work on a separate sheet of paper**. Answers to the assigned problems are included so you should check your work.

Although the Homework Administrative Regulations of the Randolph Township Schools do not permit us to "grade" your work on this summer assignment, the material in this packet will be the focus of instruction during the first few days of school and you will be tested on this material and extensions of this material during the first two weeks of classes. The material in this packet is not new; it is preparatory and review for the ensuing school year. While you might think that Calculus focuses mostly on the topics from Precalculus, you will find that our work actually focuses on Algebra techniques, which this packet focuses on. It would be to your advantage to arrive on the first day of school with this assignment completed and bring any questions that you might have regarding it. Traditionally, the degree to which a student puts the time and effort into this assignment is a good indicator of his or her success in this course.

If you have any questions, please see Ms. Lupinski or Ms. Vetrone before the end of the school year. If questions arise during the summer, please email Ms. Lupinski at: vlupinski@rtnj.org or Ms. Vetrone at: kvetrone@rtnj.org.

Good luck and have a safe and happy summer.

Sincerely,

The Calculus A Teachers

1. If $f(x) = 2x^3 - 3x^2 - x + 2$, then find $f(-1)$.
$f(-1) =$ _____

If $f(x) = 4x + 3$ and $g(x) = x^2 - 2$, then find $f(g(x))$ and $g(f(x))$.

2. $f(g(x)) =$ _____
$g(f(x)) =$ _____

3. _____ Evaluate: $\sum_{i=1}^{4}\left((i-1)^2 + (i+1)^3\right)$

4. If $y = x^2 + 5$, and $y - x = 7$, find $x$.
$x =$ _____

5. _____ If $m = \sqrt{3}$, find $m^{-3}$. Give your answer as a simplified radical.

6. Find the **DOMAIN** for each function in INTERVAL NOTATION.

| | | |
|---|---|---|
| A. $f(x) = \frac{x-6}{5}$ | B. $f(x) = \frac{6}{x+1}$ | C. $f(x) = \sqrt{x+5}$ |
| D. $f(x) = 5^x + 2$ | E. $f(x) = \frac{\sqrt{x-2}}{x-7}$ | F. $f(x) = \frac{3x}{x^2-4}$ |
| G. $f(x) = \ln(x)$ | H. $f(x) = 4\cos(x - \pi)$ | I. $y = \frac{x}{\sqrt{9-x^2}}$ |

7. **Factor** out the **GCF** and **simplify** (Hint: Do **not** expand binomials raised to powers!):

a) _____ $(x-1)^3(1) + (x+2)(3)(x-1)^2$

b) _____ $\frac{(x-1)^2(2x) - x^2(2)(x-1)}{(x-1)^4}$

8. State the 3 Pythagorean Identities.

_____ _____ _____

9. $\sin 2x =$ _____

10. PROVE the following trig identity: $\dfrac{1}{1-\sin^2 x} = 1+\tan^2 x$

11. Circle one answer: The expression $\dfrac{\sin x}{1-\cos x}$ is equivalent to: (show your work!)

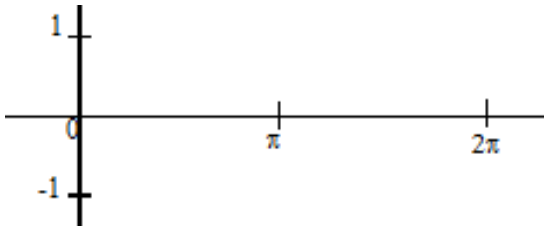
A. $\dfrac{\tan x}{1+\cos x}$ B. $\dfrac{\sin x(1+\cos x)}{(1-\cos x)^2}$ C. $\dfrac{\sin x}{1+\cos x}$ D. $\dfrac{1+\cos x}{\sin x}$

12. _____ If $\theta$ is an acute angle and $\sin\theta = \frac{3}{5}$, then what does $\sin 2\theta$ equal? Give your answer in fraction form.

13. _____ What is the EXACT VALUE of $\sin\left(\cos^{-1}\dfrac{1}{2}\right)$?

14. _____ If $\sin\theta = \frac{1}{3}$, then what does $\cos 2\theta$ equal? Give your answer in fraction form.

15. _____ What is the value of $\cos\frac{\pi}{2}$? Demonstrate your answer using a graph of the cosine function.




16. If $25^x = 5$ *and* $3^{(x+y)} = 81$, then what is the value of $y$?

$y =$ _____

17. Rewrite $f(x) = |2x - 3|$ as a piecewise function.

$f(x) =$ {

18. Use the PROPERTIES OF LOGARITHMS to expand or condense each log expression.

A. $log\ 16\sqrt[3]{x+1}$ B. $\ln\frac{x^3y}{9\sqrt{w}}$ C. $\ln x - 2\ln y + 3\ln x$ D. $\frac{1}{2}(\ln x + \ln y) - 2\ln(z+1)$

19. Circle one answer. The expression $\frac{3}{4-\sqrt{3}}$ is equivalent to which of the following: (show your work)

A. $\frac{12+3\sqrt{3}}{13}$ B. $\frac{12+3\sqrt{3}}{4+3\sqrt{3}}$ C. $\frac{12}{4+3\sqrt{3}}$ D. $12+3\sqrt{3}$

20. _____ Find the value of $(x+2)+(x+1)^{-\frac{2}{3}}$, when $x$ = 7.

21. If $x^2y + 4y^2 - 6 = -5$, find the value of $y$ when $x$ = 1. Leave the answer in simplified radical form.

$y$ = _____

22. For the function $f(x) = x^2 + 7$, evaluate and SIMPLIFY the following:

a. _____ $f(3a)$

b. _____ $f(b-1)$

c. _____ $\frac{f(x+h)-f(x)}{h}, h \neq 0.$

23. Simplify the following expressions and <u>state the domain restrictions in interval notation.</u>

a. _____ $\frac{v - v^{\frac{3}{2}}}{\sqrt{v}}$

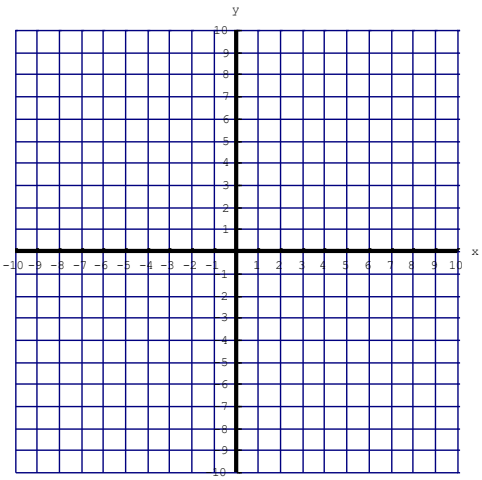
b. _____ $\frac{b^2 - 3b - 4}{b+1}$

c. _____ $\frac{(x^2y^2 - 1)+(xy-1)}{(xy-1)}$

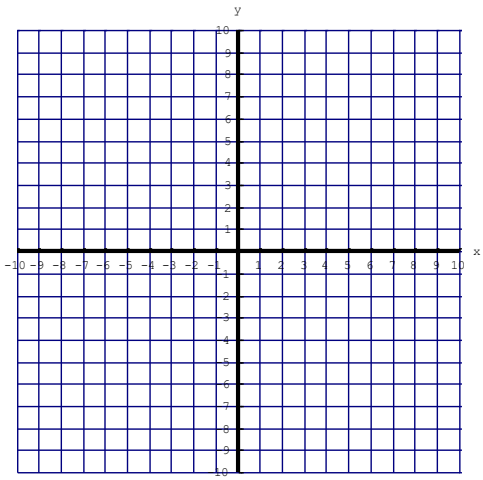
24. Explain how to find where the functions $f(x) = 4x - x^2$ *and* $g(x) = x^2$ intersect, then state the points of intersection (without the use of a graphing calculator).

25. Graph each of the following equations <u>without the use of a graphing calculator</u>. Include the domain and range of each graph in interval notation.

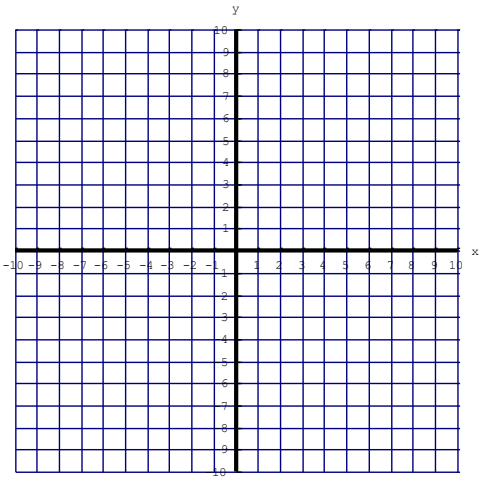
a. $y = (x-1)^2 + 3$



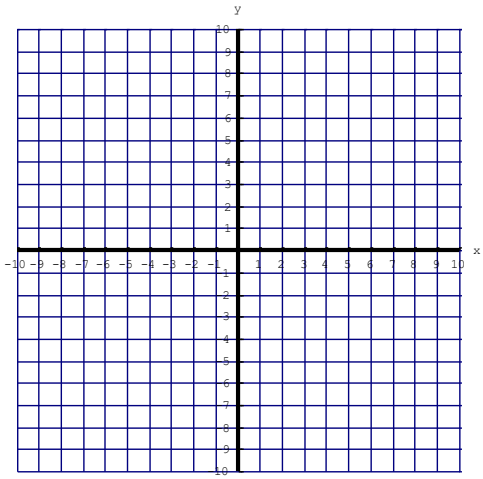
b. $x^2 + y = 36$



c. $y = \sqrt{2x+3}$



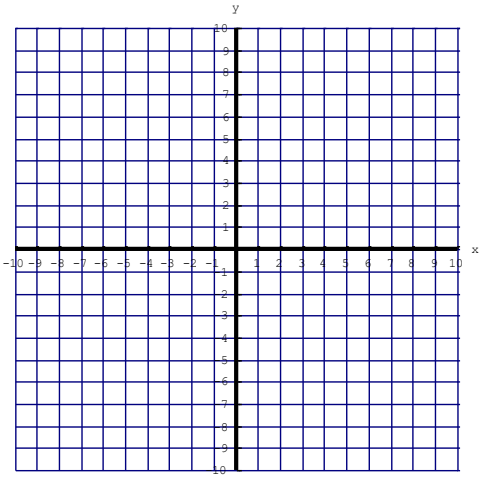
d. $y = \frac{|x|}{x}$



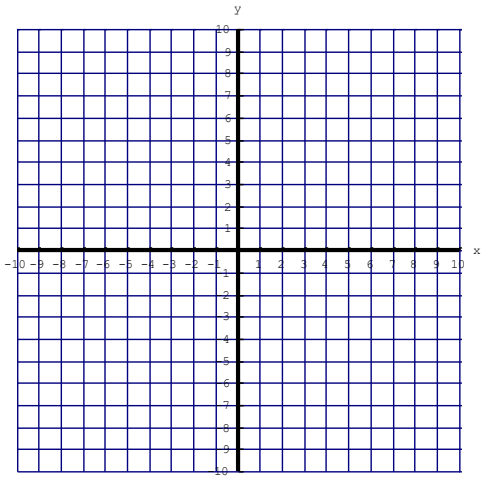
e. $y = \tan\left(x - \frac{\pi}{4}\right)$

f. $y = \sin 2x$

g. $f(x) = \begin{cases} x^2, if\ |x| \geq 2 \\ 2x, if\ |x| < 2 \end{cases}$




h. $x = y^2 - 9$



# Answer Key

1. -2 2. $4x^2 - 5;\ 16x^2 + 24x + 7$ 3. 238 4. x = 2, -1 5. $\sqrt{3}/9$

6. A. $(-\infty, \infty)$ B. $(-\infty, -1) \cup (-1, \infty)$ C. $[-5, \infty)$ D. $(-\infty, \infty)$ E. $[2,7) \cup (7, \infty)$

F. $(-\infty, -2) \cup (-2,2) \cup (2, \infty)$ G. $(0, \infty)$ H. $(-\infty, \infty)$ I. (-3, 3)

7. A. $(x-1)^2(4x+5)$ B. $\frac{-2x}{(x-1)^3}$ 8. $sin^2x + cos^2x = 1;\ 1 + cot^2x = csc^2x;\ tan^2x + 1 = sec^2x$

9. 2 sin x cos x 10. Varies 11. D 12. 24/25 13. $\sqrt{3}/2$ 14. 7/9

15. 0 16. y = 3.5 17. $f(x) = \begin{cases} -2x + 3 & x < 1.5 \\ 2x - 3 & x \geq 1.5 \end{cases}$ 18. A. $\log 16 + \frac{1}{3}\log(x+1)$

B. $3\ln x + \ln y - \ln 9 - \frac{1}{2}\ln w$ C. $\ln \frac{x^4}{y^2}$ D. $\ln \frac{\sqrt{xy}}{(z+1)^2}$ 19. A 20. 37/4

21. $y = \frac{-1 \pm \sqrt{17}}{8}$ 22. A. $9a^2 + 7$ B. $b^2 - 2b + 8$ C. $2x + h$

23. A. $v^{1/2} - v$ B. $b - 4$ C. $(xy + 2)$

• Set = & solve for x  • graph & find intersection points

24. Explain how to find where the functions $f(x) = 4x - x^2$ *and* $g(x) = x^2$ intersect, then state the points of intersection (without the use of a graphing calculator).

$4x - x^2 = x^2$

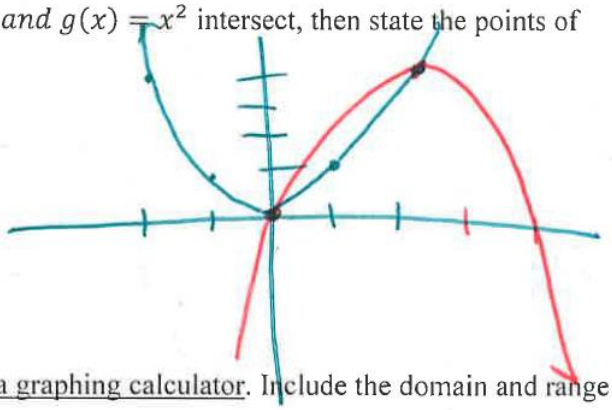
$0 = 2x^2 - 4x$

$0 = x^2 - 2x$

$0 = x(x-2)$

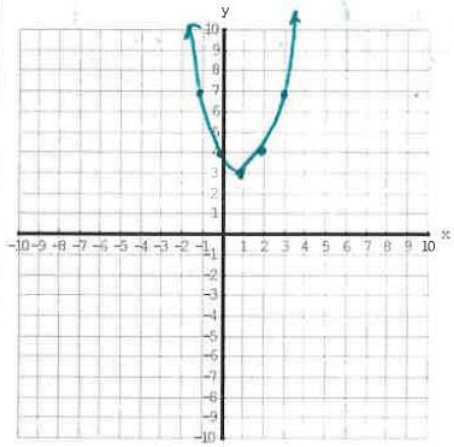
$(0,0)$

$(2,4)$



25. Graph each of the following equations <u>without the use of a graphing calculator</u>. Include the domain and range of each graph in interval notation.

a. $y = (x-1)^2 + 3$



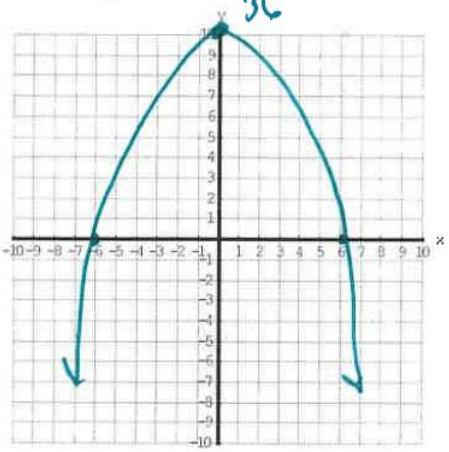
D $(-\infty, \infty)$

R $[3, \infty)$

b. $x^2 + y = 36$

$y = 36 - x^2$

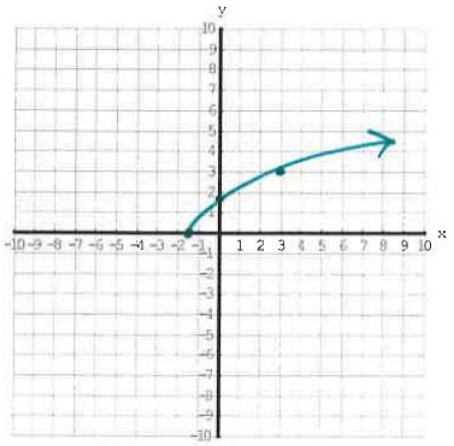
36



D $(-\infty, \infty)$

R $(-\infty, 36]$

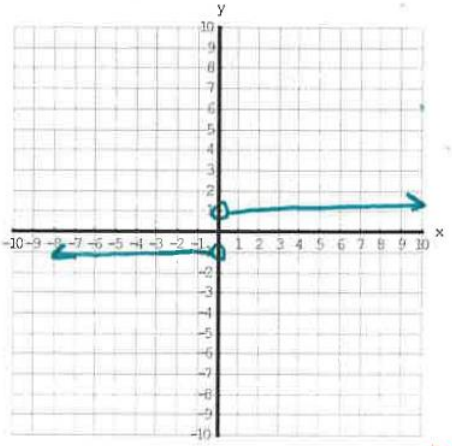
c. $y = \sqrt{2x+3}$



D $[-1.5, \infty)$

R $[0, \infty)$

d. $y = \frac{|x|}{x}$

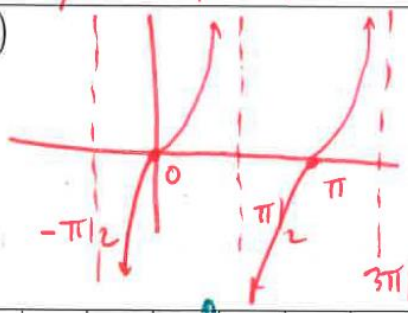


D $(-\infty, 0) \cup (0, \infty)$

R $[-1] \cup [1]$

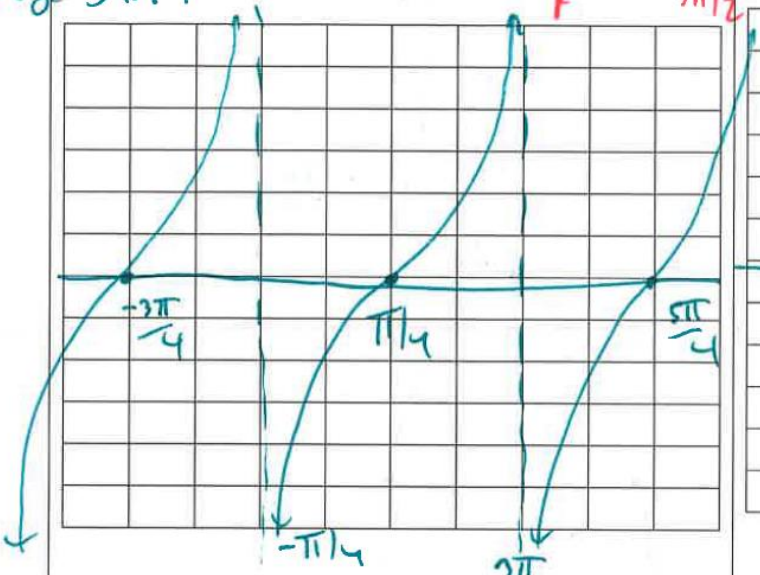
e. $y = \tan\left(x - \frac{\pi}{4}\right)$

$y = \tan x \quad P = \pi$




$PS = \frac{\pi}{4}$

Phase Shift

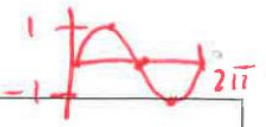



D $x \neq \frac{3\pi}{4} + \pi n$

R $(-\infty, \infty)$

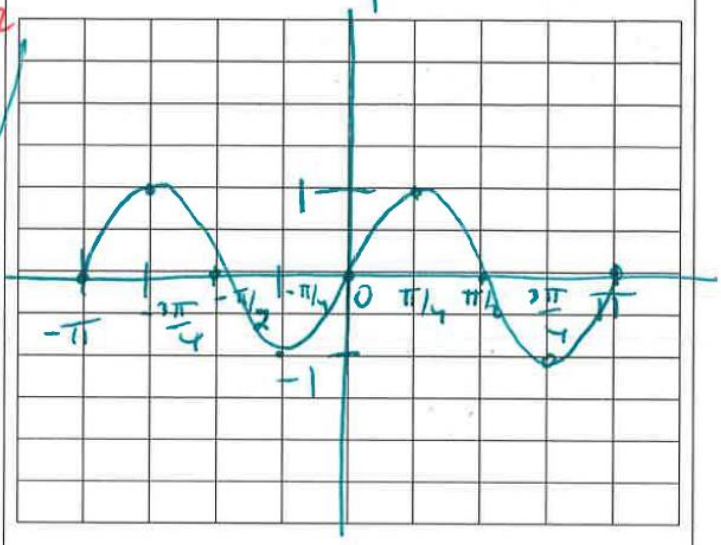
f. $y = \sin 2x$

$y = \sin x$




$P = \frac{2\pi}{B} = \frac{2\pi}{2}$

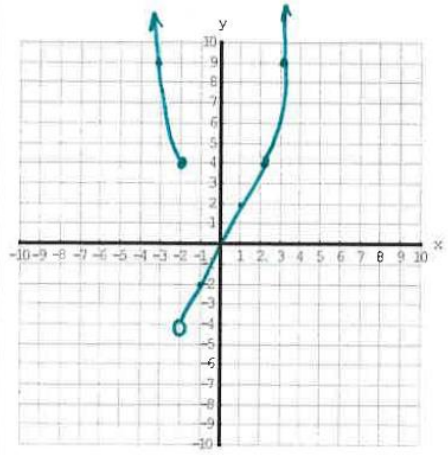
$P = \pi$




D $(-\infty, \infty)$

R $[-1, 1]$

g. $f(x) = \begin{cases} x^2, if\ |x| \geq 2 \\ 2x, if\ |x| < 2 \end{cases}$



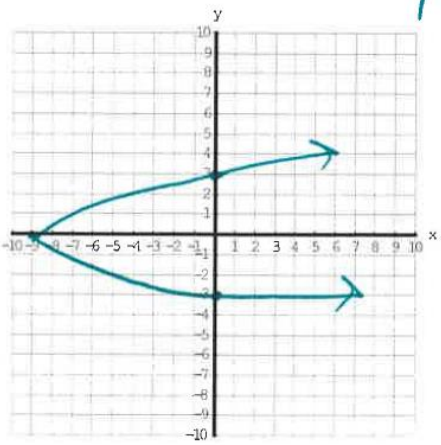

D $(-\infty, \infty)$

R $(-4, \infty)$

h. $x = y^2 - 9$

$y^2 = x + 9$

$y = \pm\sqrt{x+9}$




D $[-9, \infty)$

R $(-\infty, \infty)$