AP Calculus B/C Summer Assignment 2019-2020

Congratulations on the completion of Pre-Calculus Honors; NOW it is on to AP Calculus B/C!!! AP Calculus B/C represents the equivalent of two semesters of introductory calculus at most colleges (A/B students get the equivalent of one semester). In addition to normal quizzes, tests, benchmarks, etc. you will be taking the national AP test on the morning of

Tuesday May 5th, 2020.

As a student in Calculus, it is important that you have a thorough knowledge of skills in Algebra and Pre-Calculus. This summer assignment contains problems that are considered prerequisites for AP Calculus B/C. You should be comfortable solving problems of the types included here in order to move forward quickly with new AP Calculus B/C material in the fall. Although this packet will not be graded immediately, it is in your best interest to completely understand all topics in the assignment in order to be prepared for AP Calculus B/C!!!.

It is encouraged to stay familiar with your graphing calculator, but it is important to be able to perform many of these problems without the use of a calculator, the reason being the AP test is designed in sections where no calculator is allowed.

You will have an opportunity to ask questions on these problems and techniques in the first few days of school prior to the assessment on this material. The assessment will be by the end of the first week of school.

If you have any questions regarding this assignment, please contact me at gdouglas@rtnj.org. I will check my e-mail several times a week during the summer.

Have a wonderful summer! I look forward to another AweSUM class with you next school year.

Sincerely,

Mr. Glenn Douglas

Completed assignments should be on Separate sheets of papers in a NEATLY organized fashion showing your all work!!!

1. Find the missing value k, which gives the line a slope of $-\frac{8}{3}$. Then, write the equation of the line normal to the original line in point-slope form through the 1st ordered pair.
$\left(\frac{7}{8}, k\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$.

$k =$ ________ Equation of the normal line through $\left(\frac{7}{8}, k\right)$: ________________________

2. If $f(x) = 4x + 3$ and $g(x) = x^2 - 2,$ then find:

A. $g(f(x)) =$ _________ *B.* $f^{-1}(x) =$ __________ *C.* $f(g^{-1}(x)) =$ __________

3. Find the domain, range, asymptotes and holes in the following functions. Identify any asymptotes as vertical (V), horizontal (H) or slant (S).

Function	Domain	Range	Asymptotes	Holes
$f(x) = \dfrac{2x}{x^2 + 2x + 1}$				
$f(x) = \dfrac{3x^3 - 2x + 1}{2x^2}$				
$f(x) = \dfrac{x^2 - 5x}{4x^2 + 7x - 2}$				

4. **Factor** and **simplify** (Hint: Do **not** expand binomials raised to powers!):

A. ______________________________ $\dfrac{(x-1)^2(2x) - x^2(2)(x-1)}{(x-1)^4}$

B. ______________________________ $\dfrac{2(x+1)(x^2+2x)^2 - 2(2x+2)(x+1)^2(x^2+2x)}{(x^2+2x)^4}$

5. Find the points of intersection of the graphs $y = \sqrt{x+6}$ and $y = \sqrt{-x^2 - 4x}$.

6. Solve the equation for *x*. Show all your work.

A. $4^x + 4^{-x} = \frac{5}{2}$.   B. . $\sqrt{x^2 + 5} + x = 5$ C. $2x^3 - 3x^2 - 2x + 3 > 0$ D. $|x+2| + 2 \leq 3$

E. $xe^x = 3x$ F. $2^{x+3} = 5^{7-3x}$ G. $2\log_2 x - \log_2 1 - x = 3$

7. Let $y = 3\sin(2x - \pi) + \sqrt{2}$. Find the following and give exact values.

Amplitude: ______________ Period: ______________
Vertical shift: ____________ Horizontal shift: __________ Range: ______________

8. Find the value for each without using a calculator:

A. $\sin\left(\cos^{-1}\frac{1}{2}\right)$ B. $\sec\left(\sin^{-1}\frac{12}{13}\right)$ C. $\sin\left(\arctan\frac{12}{5}\right)$

9. Find the solutions of the equation on [0, 2π]:

A. $\sin 2\theta + \sin\theta = 0$ B. $\cos(6q + p) = 0$ C. $2\sin^2 q - \sin q = 1$

10. If $f(x) = \frac{50}{1 - e^{-x}}$, find $f^{-1}(x)$. Show all your work.

11. If x and y are real numbers, what is the domain of the function $y = \frac{x}{\sqrt{9 - x^2}}$?

12. For the function $f(x) = x^2 + 7$, evaluate and simplify the following:

A. $f(3a - 1)$ B. $\frac{f(x+h) - f(x)}{h}, h \neq 0.$

13. Simplify the following expressions and state any restrictions on the variables:

A. $\frac{v - v^{\frac{3}{2}}}{\sqrt{v}}$ B. $\frac{b^2 - 3b - 4}{b + 1}$ C. $\frac{(x^2y^2 - 1) + (xy - 1)}{(xy - 1)}$ D. $\frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{7}{x}}$

14. State the domain and range for:

A. $y = \arccos x$ Domain: __________________ Range: _________________

B. $y = \arctan x$ Domain: __________________ Range: _________________

C. $y = \log(x - 3)$ Domain: __________________ Range: _________________

D. $f(x) = \sqrt[4]{\frac{3 - x^2}{x - 5}}$ Domain: __________________ Range: _________________

15. Graph each of the equations below on a separate sheet of graph paper without the use of a graphing calculator. Include the domain and range of each graph.

A. $y=(x-1)^2+3$ B. $y=\log(x-1)-2$ C. $y=\dfrac{|x-5|}{x-5}$ D. $y=\tan\left(x-\dfrac{\pi}{4}\right)$

E. $f(x)=\begin{cases} x^2, \text{ if } |x|\geq 2 \\ 2x, \text{ if } |x|<2 \end{cases}$ F. $f(x)=\begin{cases} -x & \text{if } x<0 \\ x^2+1 & \text{if } 0<x<3 \\ -4 & \text{if } x>3 \end{cases}$

16. Determine if the sequence is arithmetic, geometric or neither. If the sequence is arithmetic, calculate the common difference and find the formula for the nth term. If it is geometric, calculate the common ratio and find the formula for the nth term.

A. 6, 24, 96, 384,... B. 1, 3, 7, 13, ... C. 4, 13, 22, 31,... D. $\frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \ldots$

17. Find the sum of the first 20 terms of the series.

A. 34 + 25 + 16 + 7 + … B. 1 + 2 + 4 + 8 + … C. $\displaystyle\sum_{i=1}^{\infty}\left(5\left(\frac{2}{3}\right)^{i-1}\right)$

18. Use sigma notation to write each sum.

A. $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\ldots$ B. 2 + 5 + 8 + 11 + … + 29

C. –1 + 2 + 7 + 14 + 23 +…. + 62 D. –2 + 2 – 2 + 2 – 2 +….

19. Evaluate: $\displaystyle\sum_{i=3}^{25}\left((i-1)^2+(i+1)^3\right)$.

20. Convert from Polar to rectangular. Leave answers as exact values.

A. $\left(-3, \frac{3\pi}{4}\right)$ B. $\left(-4, \frac{11\pi}{6}\right)$

21. Convert from rectangular to polar. Round answers to 3 decimals.

A. (5, 12) B. (-3, 5)

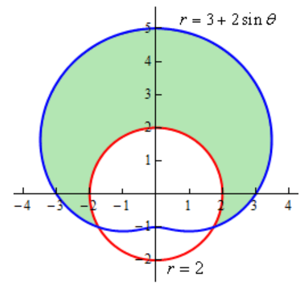
22. Change the following equations from rectangular to polar form. Solve for r.

A. $x^2 = 4y$ B. $\left(x^2 + y^2\right)^2 = ax^2y$

23. Change the following equations from polar form to rectangular form.

A. $r = 7\csc\theta$ B. $r = 16\sin\theta$

24. Determine the intersection point(s) of the polar equation $r = 3 + 2\sin\theta$ and $r = 2$



25. Parametric Equations: Sketch the curve

$$x = t^2 - 2t \qquad \text{when } -2 \le t \le 4.$$
$$y = t + 1$$

26. Eliminate the parameter to find the Cartesian equation of the parametric curve

$$x = t^2$$
$$y = 6 - 3t$$

27. Evaluate the limits analytically:

A. $\lim_{x\to 1} \dfrac{x^2 - 1}{x - 1}$

B. $\lim_{h\to 0} \dfrac{2(x+h) - 2x}{h}$

C. $\lim_{h\to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$

D. $\lim_{h\to 0} \dfrac{\tan(x+h) - \tan x}{h}$

E. $\lim_{x\to \frac{1}{3}} \dfrac{9x^2 - 1}{3x - 1}$

F. $\lim_{x\to 1} \dfrac{x - 1}{x^2 - 1}$

G. $\lim_{x\to \pi} \dfrac{1 - \cos x}{x}$

H. $\lim_{x\to 0} \left(x^2 - \dfrac{1}{x}\right)$

I. $\lim_{x\to \infty} \dfrac{4}{x}$

J. $\lim_{x\to 3} f(x)$ when $f(x) = \begin{cases} x^2 - 4x + 4, & x < 3 \\ -x^2 + 4x - 2, & x > 3 \end{cases}$

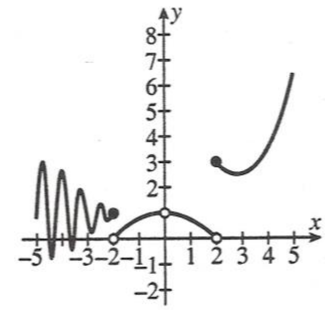
K. $\lim_{x\to -\infty} \dfrac{1 - e^x}{1 + e^x}$

L. $\lim_{x\to +\infty} \left(1 + \frac{1}{x}\right)^{-x}$

M. $\lim_{y\to -\infty} \dfrac{2 - y}{\sqrt{7 + 6y^2}}$

N. $\lim_{x\to +\infty} \left(\sqrt{x^2 + 3} - x\right)$

28. Use the graph to evaluate the limits for each of the following:

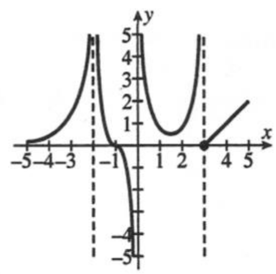


a. $\lim_{x\to -2^-} f(x) =$ b. $\lim_{x\to -2^+} f(x) =$ c. $\lim_{x\to -2} f(x) =$

d. $\lim_{x\to 2^+} f(x) =$ e. $\lim_{x\to 2^-} f(x) =$ f. $\lim_{x\to 2} f(x) =$

g. $\lim_{x\to 0^-} f(x) =$ h. $\lim_{x\to 0^+} f(x) =$ i. $\lim_{x\to 0} f(x) =$

29. Use the graph to evaluate the limits for each of the following:



a. $\lim_{x\to 2^+} f(x) =$ b. $\lim_{x\to -2^-} f(x) =$ c. $\lim_{x\to -2} f(x) =$

d. $\lim_{x\to 3^-} f(x) =$ e. $\lim_{x\to 3^+} f(x) =$ f. $\lim_{x\to 3} f(x) =$

g. $\lim_{x\to 0^+} f(x) =$ h. $\lim_{x\to 0^-} f(x) =$ i. $\lim_{x\to 0} f(x) =$

30. Find the derivative of the following equations at the given point.

A. $f(x) = \frac{1}{x^2}$ at $\left(-2, \frac{1}{4}\right)$

B. $f(x) = (x+1)^{1/2}$ at $(3, 2)$

AP Multiple Choice Questions – (YEAR(AB/BC)NUMBER)) - During the AP Test multiple choice questions are expected to take only 2 minutes per problem.

1. (1985AB29) Which of the following functions are continuous for all real numbers x?

I. $y = x^{\frac{2}{3}}$ II. $y = e^x$ III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

2. (1993AB5) If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 4}{x+2}$ when $x \neq -2$, then $f(-2) =$

(A) –4 (B) –2 (C) –1 (D) 0 (E) 2

3. (1998AB26)

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \dfrac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k =$

(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) 2 (E) 3

4. (1988BC5) Let f be the function defined by $f(x) = \begin{cases} \sin x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x < 1 \\ 2 - x & \text{if } 1 \le x < 2 \\ x - 3 & \text{if } x \ge 2 \end{cases}$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

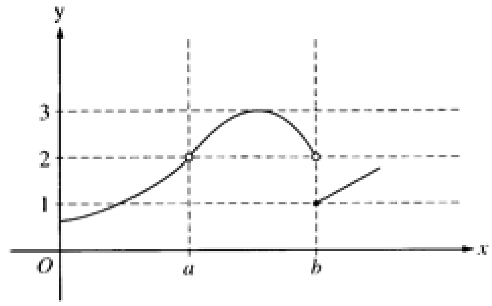
5. (1993AB2) If $f(x) = 2x^2 + 1$, then $\lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x^2}$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

6. (1997AB21) $\lim\limits_{x \to 1} \dfrac{x}{\ln x}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

7. (1997AB15)



The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$ (B) $\lim_{x\to a} f(x) = 2$ (C) $\lim_{x\to b} f(x) = 2$

(D) $\lim_{x\to b} f(x) = 1$ (E) $\lim_{x\to a} f(x)$ does not exist

8. (1998AB12) If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln x & \text{for } 2 < x \le 4 \end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) ln2 (B) ln8 (C) ln16 (D) 4 (E) nonexistent

9. (1998AB83) If $a \neq 0$, then $\lim_{x\to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent