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Randolph Township Schools

Mission Statement

*We commit to inspiring and empowering all students in Randolph Schools to reach their full potential as unique, responsible and educated members of a global society.*

Randolph Township Schools

Affirmative Action Statement

Equality and Equity in Curriculum

The Randolph Township School district ensures that the district’s curriculum and instruction are aligned to the state’s standards. The curriculum addresses the elimination of discrimination and the achievement gap, as identified by underperforming school-level AYP reports for state assessments. The curriculum provides equity in instruction, educational programs and provides all students the opportunity to interact positively with others regardless of race, creed, color, national origin, ancestry, age, marital status, affectional or sexual orientation, gender, religion, disability or socioeconomic status.

N.J.A.C. 6A:7-1.7(b): Section 504, Rehabilitation Act of 1973; N.J.S.A. 10:5; Title IX, Education Amendments of 1972
The statements represent the beliefs and values regarding our educational system. Education is the key to self-actualization, which is realized through achievement and self-respect. We believe our entire system must not only represent these values, but also demonstrate them in all that we do as a school system.

We believe:

- The needs of the child come first
- Mutual respect and trust are the cornerstones of a learning community
- The learning community consists of students, educators, parents, administrators, educational support personnel, the community and Board of Education members
- A successful learning community communicates honestly and openly in a non-threatening environment
- Members of our learning community have different needs at different times. There is openness to the challenge of meeting those needs in professional and supportive ways
- Assessment of professionals (i.e., educators, administrators and educational support personnel) is a dynamic process that requires review and revision based on evolving research, practices and experiences
- Development of desired capabilities comes in stages and is achieved through hard work, reflection and ongoing growth
Randolph Township Schools
Department of Science, Technology, Engineering, and Math

Introduction

Randolph Township Schools is committed to excellence. We believe that all children are entitled to an education that will equip them to become productive citizens of the 21st century. We believe that an education grounded in the fundamental principles of science, technology, engineering, and math (STEM) will provide students with the skills and content necessary to become future leaders and lifelong learners.

A sound STEM education is grounded in the principles of inquiry, rigor, and relevance. Students will be actively engaged in learning as they use real-world STEM skills to construct knowledge. They will have ample opportunities to manipulate materials and solve problems in ways that are developmentally appropriate to their age. They will work in an environment that encourages them to take risks, think critically, build models, observe patterns, and recognize anomalies in those patterns. Students will be encouraged to ask questions, not just the “how” and the “what” of observed phenomena, but also the “why”. They will develop the ability, confidence, and motivation to succeed academically and personally.

STEM literacy requires understandings and habits of mind that enable students to make sense of how our world works. As described in Project 2061’s Benchmarks in Science Literacy, The Standards for Technological Literacy, and Professional Standards for Teaching Mathematics, literacy in these subject areas enables people to think critically and independently. Scientifically and technologically literate citizens deal sensibly with problems that involve mathematics, evidence, patterns, logical arguments, uncertainty, and problem-solving.

AP Calculus AB

AP Calculus AB is an elective course in the STEM department for juniors and seniors who have completed Precalculus H. As an Advanced Placement course in Calculus, curriculum follows the recommendation of the College Entrance Examination Board, and is primarily concerned with developing students’ understanding of the concepts of calculus and providing experience with its methods and applications. The course emphasizes a multi-representational approach, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important. The course is intended to be challenging and demanding. Broad concepts and widely applicable methods are emphasized. Technology will be used regularly to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions discussed in Precalculus mathematics. Students who successfully complete the AB test may receive credit for one semester of college calculus. Students are encouraged to take the Advanced Placement Examination.
<table>
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<th>SUGGESTED TIME ALLOTMENT</th>
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<td>I</td>
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</table>
# ENDURING UNDERSTANDINGS

- Calculus is an extension of previously learned mathematics and yet it is fundamentally different.

- Calculus is the mathematics of change. It enables us to measure *how* quantities change (i.e. whether they are increasing or decreasing), *how fast or slow* quantities change, and *how much* quantities change.

- The limiting process lies at the heart of both differential calculus (derivatives) and integral calculus. A *derivative* is an instantaneous rate of change. It tells us whether a quantity is increasing or decreasing at a point in time and how fast or slow it is doing so. An *integral* accumulates an infinite number of small changes in a quantity based on its dynamic rate of change and tells us the net change in the quantity over a period of time.

# ESSENTIAL QUESTIONS

- In what ways is calculus similar to previously learned mathematics? In what ways does it differ?

- Historically, have societies found it important to measure change? If so, why or why not? In what ways is change measured?

- What is the limiting process and how might it be useful in measuring change?

---

## KNOWLEDGE

**Students will know:**

- An average rate of change is the change in a quantity divided by the change in time.

- An instantaneous rate of change is the limit of the average rate of change as the change in time approaches zero.

- The limit of a function $f(x)$ as values of $x$ approach a constant $a$ is intuitively defined as the $y$-value the function “appears to be heading for” as values of $x$ approach $a$ from both the left and the right.

- Infinite limits are synonymous with vertical asymptotes.

- Limits at infinity (where they exist) are synonymous with horizontal asymptotes.

## SKILLS

**Students will be able to:**

- Calculate average rates of change using the formula for free fall.

- Calculate instantaneous rates of change using the formula for free fall.

- Evaluate limits
  - Numerically (with a graphing calculator),
  - Graphically, and
  - Algebraically (using factor and cancel, rationalization, and simplification).

- Use one-sided limits to describe the behavior of functions around vertical asymptotes.

- Use limits at infinity to describe the end behavior of functions.

## CCSS

- HSN.RN.1,2
- HSN.Q.3
- HSN.CN.1
- HSA.SSE.1,2,3
- HSA.APR.1-7
- HSA.CED.1.2
- HSA.REI.1-4,6,7,10
- HSF.IF.1-5,7-9
- HSF.BF.1-5
- HSF.LE.1-5
- HSS.TF.1-4,8,9
- SMP.1-8
- ELA-Literacy.WHST.11-12.2
- ELA-Literacy.WHST.11-12.4
- ELA-Literacy.WHST.11-12.6
- ELA-Literacy.WHST.11-12.8
- ELA-Literacy.RST.11-12.3
Limits fail to exist at jump discontinuities, vertical asymptotes (infinite discontinuities), or oscillating discontinuities. Limits do exist at point (removable) discontinuities.

Limits do not depend on the value of a function at a point – i.e., limits might exist where a function is not defined and might not exist where a function is defined.

A function is continuous at a point if a limit exists at that point and the value of the function there has the same value as the limit.

The Intermediate Value Theorem is a consequence of continuity. It is useful in determining whether or not roots of functions (and later, critical values and/or points of inflection) exist on a given interval.

Formulas for the Limit Definition of the Derivative, Derivative at a Point, and the Symmetric Difference Quotient.

Where functions are not differentiable (i.e., at any discontinuity, at sharp turns, and where vertical tangents exist)

<p>| <strong>Limits fail to exist at jump discontinuities, vertical asymptotes (infinite discontinuities), or oscillating discontinuities. Limits do exist at point (removable) discontinuities.</strong> |
| <strong>Limits do not depend on the value of a function at a point – i.e., limits might exist where a function is not defined and might not exist where a function is defined.</strong> |
| <strong>A function is continuous at a point if a limit exists at that point and the value of the function there has the same value as the limit.</strong> |
| <strong>The Intermediate Value Theorem is a consequence of continuity. It is useful in determining whether or not roots of functions (and later, critical values and/or points of inflection) exist on a given interval.</strong> |
| <strong>Formulas for the Limit Definition of the Derivative, Derivative at a Point, and the Symmetric Difference Quotient.</strong> |
| <strong>Where functions are not differentiable (i.e., at any discontinuity, at sharp turns, and where vertical tangents exist)</strong> | <strong>Identify graphically and algebraically the different types of discontinuities that might exist in the graph of a function and determine whether or not limits exist at those places.</strong> |
|  | <strong>Sketch the possible graphs of a function given certain criteria, such as different types of discontinuities and determine whether limits exist at those places.</strong> |
|  | <strong>Analyze functions graphically and algebraically to determine the intervals on which they are continuous and for what values of x they are discontinuous. Identify the types of discontinuities that exist if any.</strong> |
|  | <strong>State the hypothesis and the conclusion of the Intermediate Value Theorem (IVT) and given two points, use the IVT to determine if a function is guaranteed to have one or more roots on the interval determined by those points.</strong> |
|  | <strong>Calculate derivatives of a function at various points. Derive expressions from which derivatives can be calculated at any point. Demonstrate why graphing calculators may give erroneous results when calculating certain derivatives.</strong> |
|  | <strong>Identify graphically and algebraically where functions fail to be differentiable.</strong> |</p>
<table>
<thead>
<tr>
<th>SUGGESTED TIME ALLOTMENT</th>
<th>CONTENT – UNIT OF STUDY</th>
<th>SUPPLEMENTAL UNIT RESOURCES</th>
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<tr>
<td>4 weeks</td>
<td>Unit I – Limits, Continuity, and Differentiability</td>
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<td>o Understanding the Limit Process</td>
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<td>o Evaluating Limits Graphically, Numerically and Algebraically</td>
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<td>o Evaluating Limits of Trigonometric Functions</td>
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<td>o One-Sided Limits</td>
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<td>o Infinite Limits</td>
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<td>o Limits at Infinity</td>
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<td>o Where Limits Fail to Exist</td>
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<td>o Limits and Continuity</td>
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<td>o The 4 Types of Discontinuities</td>
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<td>o Continuity and the Intermediate Value Theorem</td>
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<td>o The Limit Definition of the Derivative</td>
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<td>o Differentiability</td>
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<td></td>
<td>For AP Exam Free Response Questions on Limits, Continuity, Differentiability, see ap13_calc_fraq_coll_index (FRQ Cross-Reference Index) in the AP Calculus Resource Binder</td>
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<td></td>
<td>For AP Exam Multiple Choice Questions on Limits, Continuity, Differentiability, see ap06_calc_mindx_incl_2008 (Multiple Choice Cross-Reference Index) in the AP Calculus Resource Binder</td>
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<td>Limits You Tube Playlist (29 Videos)</td>
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<td></td>
<td><a href="https://www.youtube.com/playlist?list=PL4F484F857E96EDD9">https://www.youtube.com/playlist?list=PL4F484F857E96EDD9</a></td>
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<td>For suggestions on emphases, classroom activities, labs, projects, and writing assignments see:</td>
<td></td>
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<tr>
<td></td>
<td>• Teaching AP Calculus (McMullin), Ch. 5</td>
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<td>• Instructor’s Manual Calculus 8th Edition (Anton et al), Ch. 2</td>
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<td>• Teacher’s Guide with Answers: Calculus (Finney, et al), Ch. 2</td>
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<td><strong>Learning By Discovery: A Lab Manual for Calculus (Solow), Lab 2-5</strong></td>
<td><strong>Discovering Calculus with Graphing Calculators (McCarter), Projects 5-10</strong></td>
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<tr>
<td><strong>Calculus Calculator Labs Workbook (Albert et al), Lab 3, 4</strong></td>
<td><strong>Calculus Activities: TI-83 Plus, TI-84 Plus Families (Antinone et al), Activity 1-5</strong></td>
<td></td>
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<tr>
<td><strong>Calculus Problems for a New Century (Fraga), Section II.1-5</strong></td>
<td><strong>Connecting Mathematics with Science: Experiments for Calculus (Lublinskaya), Ch. 1-2</strong></td>
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<tr>
<td><strong>A Watched Cup Never Cools (Kamischke), Lab Activity 2; Writing Assignment 3</strong></td>
<td><strong>Calculation AP Edition 9th Edition (Larson et al), p. 90, Section Project: Graphs and Limits of Trigonometric Functions</strong></td>
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<tr>
<td><strong>Single-Variable Calculus: Concepts &amp; Contexts with Vector Functions (Stewart) p. 155, Writing Project: Early Methods for Finding Tangents</strong></td>
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</table>

**Sample AP-Style unit assessments:**

- **Calculus AP Edition 9th Edition (Larson et al), AP1-1, AP Test Prep Exercises**
- **Rogawski's Calculus for AP Early Transcendentals 2nd Edition (Rogawski et al), Preparing for the AP Exam AP2-1**

**Problem Solving Extensions and Enrichment:**

- **Single-Variable Calculus: Concepts & Contexts with Vector Functions (Stewart) p. 179, Focus on Problem Solving**
After developing the limit process to measure rates of change, mathematicians developed algebraic techniques that could be used to find derivatives of every type of function or mathematical expression. Because of this, calculus became an inventory of mathematical techniques and strategies used to analyze change quantitatively.

### ENDURING UNDERSTANDINGS

After developing the limit process to measure rates of change, mathematicians developed algebraic techniques that could be used to find derivatives of every type of function or mathematical expression. Because of this, calculus became an inventory of mathematical techniques and strategies used to analyze change quantitatively.

### ESSENTIAL QUESTIONS

- When groundbreaking new ideas are discovered, what is it about the human spirit that drives us to develop rigorous systems of knowledge around them?

### KNOWLEDGE

**Students will know:**

Although derivatives are defined by the limit process, mathematicians have formulated proven shortcuts that make the differentiation process much simpler.

Derivatives for the six basic trigonometric functions are generated using special trigonometric limits and the chain rule.

A tangent line is the line tangent to a curve at a point. A normal line is perpendicular to a tangent line at the point of tangency.

A higher-order derivative is the derivative (rate of change) of a derivative. This can be an iterative process.

Assumptions can be made about the graph of a derivative if the graph of the function is known.

### SKILLS

**Students will be able to:**

Calculate derivative values and/or generate derivative functions for polynomial and rational expressions using the following shortcut rules:

- The Constant Rule
- The Power Rule
- The Constant Multiple Rule
- The Sum and Difference Rule
- The Product Rule
- The Quotient Rule
- The Chain Rule

Generate derivative rules for the six basic trigonometric functions using special trigonometric limits and the chain rule.

Generate equations of tangent lines and normal lines.

Compute higher-order derivatives (derivatives of derivatives).

Identify and/or sketch the graph of a function’s derivative based on the graph of the function itself, or on a list of criteria about the function.

### CCSS

- HSN.RN.1,2
- HSN.Q.3
- HSN.CN.1
- HSA.SSE.1,2,3
- HSA.APR.1-4,6,7
- HSA.CED.1,2
- HSA.REI.1-4,6,7,10
- HSF.IF.1-5,7-9
- HSF.BF.1-3,5
- HSF.LE.1-5
- HSS.TF.1-9
- HSG.SRT.7,8
- HSG.C.4
- HSG.GPE.5
- SMP.1-8
- ELA-Literacy.WHST.11-12.2
- ELA-Literacy.WHST.11-12.4
- ELA-Literacy.WHST.11-12.6
- ELA-Literacy.WHST.11-12.8
- ELA-Literacy.RST.11-12.3
- ELA-Literacy.RST.11-12.4
- ELA-Literacy.RST.11-12.5
- ELA-Literacy.RST.11-12.10
<table>
<thead>
<tr>
<th>Assumptions can be made about the graph of a function if the graph of its derivative is known.</th>
<th>Identify and/or sketch the graph of a function based on the characteristics of the function’s derivative, or on a list of criteria about the function.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derivatives can be calculated even when a relationship cannot be defined or written explicitly as a function of a single variable.</td>
<td>Apply the process of implicit differentiation to compute first and second derivatives of functions that are not defined or written explicitly as a function of a single variable.</td>
</tr>
</tbody>
</table>
| Derivatives of exponential, logarithmic, inverse trigonometric and general inverse functions can be calculated using the process of implicit differentiation. | Apply the process of implicit differentiation to compute derivative values and/or generate derivative functions for the following function families:  
- Exponential  
- Logarithmic  
- Inverse Trigonometric  
- All Other Inverse Functions |
| Derivatives of functions with variable bases but constant exponents can be calculated using the power rule. Derivatives of functions with constant bases but variable exponents are calculated using the exponential rule. Derivatives of functions with variable bases and variable exponents are calculated using logarithmic differentiation. | Apply the technique of logarithmic differentiation to compute derivatives of functions with variable bases and variable exponents, as well as other complex expressions. |
### Randolplh Township School District
#### Unit II - Curriculum Pacing Chart
#### AP Calculus AB

<table>
<thead>
<tr>
<th>Suggested Time Allotment</th>
<th>Content – Unit of Study</th>
<th>Supplemental Unit Resources</th>
</tr>
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<tbody>
<tr>
<td>6 weeks</td>
<td>Unit II – Differentiation&lt;br&gt;  - Constant Rule&lt;br&gt;  - Product Rule&lt;br&gt;  - Constant Multiple Rule&lt;br&gt;  - Product Rule&lt;br&gt;  - Quotient Rule&lt;br&gt;  - Derivatives of Trigonometric Functions&lt;br&gt;  - Chain Rule&lt;br&gt;  - Implicit Differentiation&lt;br&gt;  - Derivatives of Exponential, Logarithmic, Inverse Trigonometric, and general inverse functions&lt;br&gt;  - Logarithmic Differentiation</td>
<td>For AP Exam Free Response Questions on Differentiation, see ap13_calc_fraq_coll_index (FRQ Cross-Reference Index) in the AP Calculus Resource Binder&lt;br&gt; For AP Exam Multiple Choice Questions on Differentiation, see ap06_calc_mcindx_incl_2008 (Multiple Choice Cross-Reference Index) in the AP Calculus Resource Binder&lt;br&gt; Calculus Formulas Review <a href="https://www.youtube.com/watch?v=dDHhevNwDTo&amp;list=PL4F6AB1A2CEB01DC9&amp;index=1">Link</a>&lt;br&gt; Calculus I in 20 Minutes <a href="https://www.youtube.com/watch?v=EX_is9LzFSY&amp;list=PL4F6AB1A2CEB01DC9&amp;index=2">Link</a>&lt;br&gt; Derivative Rules You Tube Playlist (19 Videos) <a href="https://www.youtube.com/playlist?list=PL5AD1AD33C7F4886C">Link</a>&lt;br&gt; Chain Rule You Tube Playlist (18 Videos) <a href="https://www.youtube.com/playlist?list=PLA771F81A47A47A26">Link</a>&lt;br&gt; Implicit Differentiation You Tube Playlist (9 Videos) <a href="https://www.youtube.com/playlist?list=PL57CC5225464D63DF">Link</a></td>
</tr>
</tbody>
</table>
Derivatives of Exponential, Logarithmic, Inverse Trigonometric, and Inverse Functions You Tube Playlist (12 Videos)
https://www.youtube.com/playlist?list=PL3yTFasMGvQvT8KiGQTlmJsmX9k2jiRg

Multiple Choice Practice: Derivatives of Exponential, Logarithmic, Inverse Trigonometric, and Inverse Functions You Tube Playlist (29 Videos)
https://www.youtube.com/playlist?list=PL3yTFasMGvQt2F618MJaAcZeY_pD7YMj5

Derivatives Review You Tube Playlist (11 Videos)
https://www.youtube.com/playlist?list=PLF6E3B4FE4191AEDC

For suggestions on emphases, classroom activities, labs, projects, and writing assignments see:

- Teaching AP Calculus (McMullin), Ch. 6-9
- Instructor's Manual Calculus 8th Edition (Anton et al), Ch. 3, 4
- Teacher’s Guide with Answers: Calculus (Finney, et al), Ch. 3
- Learning By Discovery: A Lab Manual for Calculus (Solow), Lab 6, 7
- Discovering Calculus with Graphing Calculators (McCarter), Projects 11, 12
- Calculus Calculator Labs Workbook (Albert et al), Lab 5, 6
- Calculus Activities: TI-83 Plus, TI-84 Plus Families (Antinone et al), Activity 6-8
- Calculus Problems for a New Century (Fraga), Section II.6-9
- Connecting Mathematics with Science: Experiments for Calculus (Lublinskaya), Ch. 5, 10
- A Watched Cup Never Cools (Kamischke), Lab Activity 3, 4, 8, 10; Writing Assignment 5, 8-9
- Single-Variable Calculus: Concepts & Contexts with Vector Functions (Stewart) p. 231, Laboratory Project: Bezier Curves

Sample AP-Style unit assessments:

- Rogawski's Calculus for AP Early Transcendentals 2nd Edition (Rogawski et al), Preparing for the AP Exam AP3-1

Problem Solving Extensions and Enrichment:

- Single-Variable Calculus: Concepts & Contexts with Vector Functions (Stewart) p. 258, Focus on Problem Solving
### ENDURING UNDERSTANDINGS
Starting from the 17th century, mathematicians and scientists built on Newton’s and Leibniz’s new way of using the limiting process to understand and measure change, by applying the tools of calculus to a broad range of phenomena, including those from the mathematical, physical, and social sciences, as well as from finance.

### ESSENTIAL QUESTIONS
- Have the applications of calculus been exhausted?
- How might this way of looking at the world still provide beneficial applications?
- Are there other ways of looking at natural, social, and financial phenomena that may prove to be just as productive as calculus has been?

### KNOWLEDGE

**Students will know:**

- A critical value is a point on a graph where the derivative is zero or does not exist.

The Extreme Value Theorem (EVT) guarantees the existence of absolute extrema for continuous functions on closed intervals. The Candidate Test states that absolute extrema occur at critical values or endpoints when the hypothesis of the EVT is satisfied.

The Mean Value Theorem (MVT) guarantees the existence of at least one point on a graph where the instantaneous rate of change is equal to the average rate of change over an interval on which the function is continuous (on the closed interval) and differentiable (on the open interval).

A function has a relative maximum at a point where its first derivative changes from positive to negative (i.e. when the function changes from increasing to decreasing). A function has a relative minimum at a point where its first derivative changes from negative to positive (i.e. when the function changes from decreasing to increasing).

### SKILLS

**Students will be able to:**

- Identify critical values (graphically and algebraically) for functions representing all function families.

Determine whether a curve segment satisfies the hypothesis of the EVT and then use the Candidate Test to identify absolute extrema.

Determine whether a curve segment satisfies the hypothesis of the MVT and then identify the point(s) at which the instantaneous rate of change is equal to the average rate of change.

Identify relative maxima and relative minima (graphically and algebraically) for functions representing all function families.

### CCSS

- HSN.RN.1,2
- HSN.Q.1-3
- HSN.CN.1
- HSA.SSE.1,2,3
- HSA.APR.1-4,6,7
- HSA.CED.1-4
- HSA.REI.1-4,6,7,10,11
- HSF.IF.1,3-9
- HSF.BF.1,3-5
- HSF.LE.1-5
- HSS.TF.1-9
- HSG.SRT.2,5-8
- HSG.C.4
- HSG.GPE.5
- HSG.GMD.3
- HSG.MG.1-3
- SMP.1-8

- ELA-Literacy.WHST.11-12.2
- ELA-Literacy.WHST.11-12.4
- ELA-Literacy.WHST.11-12.6
- ELA-Literacy.WHST.11-12.8
- ELA-Literacy.RST.11-12.3
- ELA-Literacy.RST.11-12.4
- ELA-Literacy.RST.11-12.5
- ELA-Literacy.RST.11-12.10
<table>
<thead>
<tr>
<th>A function is concave up when its second derivative is positive. It is concave down when its second derivative is negative. A point of inflection is a point on a graph where the second derivative changes sign (i.e. the graph of the function changes concavity)</th>
<th>Identify points of inflection (graphically and algebraically) for functions representing all function families</th>
</tr>
</thead>
<tbody>
<tr>
<td>The tools of pre-calculus and differential calculus can be used to accurately graph the salient features of a function without graphing technology.</td>
<td>Sketch a graph of function using roots, intervals on which the function is positive and/or negative, absolute and relative</td>
</tr>
<tr>
<td>The tools of differential calculus can be used to analyze rectilinear motion, i.e. the position of an object, its velocity and direction of motion, where the object turns around, when it is at rest, and its acceleration.</td>
<td>Given a function representing the position of an object determine the position, velocity and direction of motion, where the object turns around, when it is at rest, and its acceleration.</td>
</tr>
<tr>
<td>The tools of differential calculus can be used to solve optimization problems, i.e. real-world applications where maximizing or minimizing quantities subject to various constraints is sought</td>
<td>Given a real-world or mathematical quantity subject to various constraints, apply the tools of differential calculus to identify optimal (maximum or minimum) values and where they occur.</td>
</tr>
<tr>
<td>Tangent lines and differentials can be used to approximate values of functions on intervals near the point of tangency.</td>
<td>Use tangent lines and differentials to approximate values of a function on an interval near the point of tangency. Also determine the error introduced by such an approximation.</td>
</tr>
<tr>
<td>Newton’s Method is an iterative process that uses the tools of differential calculus to approximate roots of functions when easier strategies cannot be used. (Optional)</td>
<td>Use Newton’s Method to approximate roots of functions when easier strategies cannot be used. (Optional)</td>
</tr>
<tr>
<td>L’Hospital’s Rule is an iterative process that uses the tools of differential calculus to evaluate limits of expressions with an indeterminate form.</td>
<td>Use L’Hospital’s Rule to evaluate limits of expressions with a variety of indeterminate forms.</td>
</tr>
<tr>
<td>The tools of differential calculus can be used to solve a class of real-world applications known as “related-rates” problems, in which multiple quantities are changing at the same time but some of the rates of change are unknown (but related mathematically).</td>
<td>Solve a variety of related-rates problems based on dynamic geometrical systems.</td>
</tr>
</tbody>
</table>
### Unit III - Applications of the Derivative

<table>
<thead>
<tr>
<th>SUGGESTED TIME ALLOTMENT</th>
<th>CONTENT – UNIT OF STUDY</th>
<th>SUPPLEMENTAL UNIT RESOURCES</th>
</tr>
</thead>
</table>
| 5 weeks                  | Unit III – Applications of the Derivative
  - Absolute Extrema (Extreme Value Theorem)
  - Critical Values and Local Extrema
  - Mean Value Theorem
  - Points of Inflection and Concavity
  - Curve Sketching
  - Modelling and Optimization
  - Linear Approximation and Differentials
  - Newton’s Method (Optional)
  - L’Hopital’s Rule
  - Related Rates
  - Anti-derivatives
  - Rectilinear Motion: Position, Velocity and Acceleration | For AP Exam Free Response Questions on Applications of the Derivative, see ap13_calc_frq_coll_index (FRQ Cross-Reference Index) in the AP Calculus Resource Binder
For AP Exam Multiple Choice Questions on Applications of the Derivative, see ap06_calc_mcindx_incl_2008 (Multiple Choice Cross-Reference Index) in the AP Calculus Resource Binder
Extrema: A Curriculum Module for AP Calculus
Calculus Formulas Review
https://www.youtube.com/watch?v=dDHhevNwDTo&list=PL4F6AB1A2CEB01DC9&index=1
Calculus I in 20 Minutes
https://www.youtube.com/watch?v=EX_is9LzFSY&list=PL4F6AB1A2CEB01DC9&index=2
Related Rates You Tube Playlist (11 Videos)
http://www.youtube.com/playlist?list=PLA6E4D829EAF0105C
Applications of Derivatives You Tube Playlist (8 Videos)
http://www.youtube.com/playlist?list=PLE5194FABE9F6DC85 |
Applications of Derivatives FRQ Slideshow with Video Links  
http://www.slideshare.net/jaflint718/ap-calculus-applications-of-derivatives-frq

Related Rates FRQ Slideshow with Video Links  
http://www.slideshare.net/jaflint718/ap-calculus-related-rates-frq

Related Rates MC Slideshow with Video Links  
http://www.slideshare.net/jaflint718/ap-calculus-related-rates-mc

For suggestions on emphases, classroom activities, labs, projects, and writing assignments see:

- Teaching AP Calculus (McMullin), Ch. 10
- Instructor’s Manual Calculus 8th Edition (Anton et al), Ch. 5
- Teacher’s Guide with Answers: Calculus (Finney, et al), Ch. 4
- Learning By Discovery: A Lab Manual for Calculus (Solow), Lab 8, 9, 10
- Discovering Calculus with Graphing Calculators (McCarter), Projects 11-23
- Calculus Calculator Labs Workbook (Albert et al), Lab 7, 8
- Calculus Activities: TI-83 Plus, TI-84 Plus Families (Antinone et al), Activity 9-12
- Calculus Problems for a New Century (Fraga), Section II.10-11, III.1-3
- Problems for Student Investigation (Jackson et al), Ch. I, p. 9-56
- Connecting Mathematics with Science: Experiments for Calculus (Lublinskaya), Ch. 6
- A Watched Cup Never Cools (Kamischke), Lab Activity 7; Writing Assignment 4
| Single-Variable Calculus: Concepts & Contexts with Vector Functions (Stewart) p. 277, Laboratory Project: The Calculus of Rainbows |
| Single-Variable Calculus: Concepts & Contexts with Vector Functions (Stewart) p. 232, Writing Project: The Origins of L'Hospital’s Rule |

Sample AP-Style unit assessments:

| Rogawski's Calculus for AP Early Transcendentals 2nd Edition (Rogawski et al), Preparing for the AP Exam AP4-1 |

Problem Solving Extensions and Enrichment:

| Single-Variable Calculus: Concepts & Contexts with Vector Functions (Stewart) p. 339, Focus on Problem Solving |
**ENDURING UNDERSTANDINGS**

Historians posit that the fundamental theorem of calculus (which connects the differential and integral branches of calculus) was discovered by Newton and Leibniz simultaneously and independent of each other. This is possible since many of the ideas that calculus was built upon were well known to both, and were subjects of common research among mathematicians and scientists of their day. Newton famously said, “If I have seen further, it is by standing on the shoulders of Giants.” This provides an important perspective for viewing groundbreaking new discoveries, i.e. to see them as much a product of their epoch as of the individual “geniuses” who formalized them.

**ESSENTIAL QUESTIONS**

- What implications can be derived from the simultaneous and independent discovery of calculus?
- What preparation and mindset enabled Newton and Leibniz out of so many to take advantage of the ideas that were “in the air” at their time?
- What ideas are “in the air” today?

<table>
<thead>
<tr>
<th>KNOWLEDGE</th>
<th>SKILLS</th>
<th>CCSS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Students will know:</strong></td>
<td><strong>Students will be able to:</strong></td>
<td>HSN.RN.1,2 HSN.Q.1-3 HSN.CN.1 HSA.SSE.1.2,3 HSA.APR.1-4,6,7 HSA.CED.1-2 HSA.REL.1-4,6,7,10,11 HSF.IF.1-3-9 HSF.BF.1-3-5 HSF.LE.1-5 HSS.TF.1-3-9 HSG.SRT.2,5-8 HSG.C.5 HSG.GMD.1-4 HSG.MG.1-3 SMP.1-8 ELA-Literacy.WHST.11-12.2 ELA-Literacy.WHST.11-12.4 ELA-Literacy.WHST.11-12.6 ELA-Literacy.WHST.11-12.8 ELA-Literacy.RST.11-12.3 ELA-Literacy.RST.11-12.4 ELA-Literacy.RST.11-12.5 ELA-Literacy.RST.11-12.10</td>
</tr>
<tr>
<td>A definite integral is analogous to the area between a curve and the x-axis. A Riemann Sum is a sum of rectangular areas that can be used to approximate definite integrals and calculate them exactly when an infinite number of infinitesimally thin rectangles is used. In practice, left, right, and midpoint approximations are used to illustrate this process and serve as rough estimates of definite integrals. The trapezoidal rule (which can also be viewed as the average of the left and right rectangular approximation methods) is another common way of estimating the value of a definite integral. The Fundamental Theorem of Calculus (FTC) Part I provides an algebraic strategy to evaluate definite integrals. The Mean Value Theorem for Integrals (MVTI) is a consequence of the FTC Pt. I. When a function is continuous on a closed interval, MVTI guarantees the existence of a point on the graph at which the area of the rectangle (whose height is given by the point and whose width is determined by the endpoints of the interval) is equal to the area under the curve over that interval. The y-coordinate of this point is called the average value of the function on that interval.</td>
<td>Use left, right, and midpoint rectangular approximation to estimate definite integrals and the area between a curve and the x-axis. Use the trapezoidal rule to estimate definite integrals and the area between a curve and the x-axis. Evaluate definite integrals and calculate the area between a curve and the x-axis using the FTC Part I. Use the FTC Pt. I to determine the average value of a function as guaranteed by the MVTI.</td>
<td></td>
</tr>
<tr>
<td>The Fundamental Theorem of Calculus (FTC) Part II provides an algebraic strategy to find the derivative of an integral defined function</td>
<td>Determine the derivative of an integral defined function using the FTC Part II.</td>
<td></td>
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<tr>
<td>There are many Free-Response Questions on previously released AP Calculus tests that investigate the FTC Pt. I and Pt. 2 from both a conceptual and applied perspective.</td>
<td>Apply the FTC Parts I and II to solve a variety of AP Calculus Free-Response Questions.</td>
<td></td>
</tr>
<tr>
<td>Definite integrals can be used to represent and solve for the area between a curve and the x-axis or between a curve and the y-axis.</td>
<td>Construct and evaluate definite integrals and use them to calculate the area between a curve and the x-axis or between a curve and the y-axis.</td>
<td></td>
</tr>
<tr>
<td>Definite integrals can be used to calculate the area between two curves.</td>
<td>Construct and evaluate definite integrals to calculate the area between two curves.</td>
<td></td>
</tr>
<tr>
<td>Definite integrals can be used to calculate the volume of solids conceptualized as a curve segment revolved around either the x-axis or y-axis. When revolved around the x-axis, cross-sections of these solids primarily take the shape of disks and washers.</td>
<td>Construct definite integrals and use them to calculate volumes of revolution about the x-axis (or lines parallel to the x-axis) or y-axis using the technique of disks and washers.</td>
<td></td>
</tr>
<tr>
<td>Definite integrals can be used to calculate the volume of solids conceptualized as a curve segment revolved around either the x-axis or y-axis. When revolved around the y-axis, layers of these solids primarily take the shape of cylindrical shells.</td>
<td>Construct definite integrals and use them to calculate volumes of revolution about the x-axis (or lines parallel to the x-axis) or y-axis using the technique of cylindrical shells.</td>
<td></td>
</tr>
<tr>
<td>Definite integrals to calculate the volume of solids with known cross-sections perpendicular to the x-axis or y-axis.</td>
<td>Construct definite integrals and use them to calculate the volumes of solids with known cross-sections perpendicular to the x-axis or the y-axis.</td>
<td></td>
</tr>
<tr>
<td>Definite integrals can be used to determine the length of a curve segment between two points.</td>
<td>Construct definite integrals and use them to determine the length of curve segment between two points.</td>
<td></td>
</tr>
<tr>
<td>SUGGESTED TIME ALLOTMENT</td>
<td>CONTENT – UNIT OF STUDY</td>
<td>SUPPLEMENTAL UNIT RESOURCES</td>
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</table>
| 5 weeks                  | Unit IV – Definite Integrals  
  o Rectangular Approximation (Riemann Sums) and the Trapezoidal Rule  
  o Properties of Definite Integrals  
  o The Fundamental Theorem of Calculus Pt. I  
  o Mean Value Theorem for Integrals (Average Value of a Function)  
  o The Fundamental Theorem of Calculus Pt. II  
  o Conceptual Exercises on the FTC  
  o Applications of the FTC  
  o Area and Volume | For AP Exam Free Response Questions on Definite Integrals, see ap13_calc_frq_coll_index (FRQ Cross-Reference Index) in the AP Calculus Resource Binder  
 For AP Exam Multiple Choice Questions on Definite Integrals, see ap06_calc_mcindx_incl_2008 (Multiple Choice Cross-Reference Index) in the AP Calculus Resource Binder  
 Motion: A Curriculum Module for AP Calculus  
 http://apcentral.collegeboard.com/apc/public/repository/AP_CurricModCalculusMotion.pdf  
 Fundamental Theorem of Calculus: A Curriculum Module for AP Calculus  
 Functions Defined by Integrals: A Curriculum Module for AP Calculus  
 Reasoning from Tabular Data: A Curriculum Module for AP Calculus  
| **Exploring the FTC from Numerical and Graphical Points of View with the TI-89** |
| **Volumes of Solids of Revolution: A Curriculum Module for AP Calculus** |
| **Calculus Formulas Review** |
| https://www.youtube.com/watch?v=dDHhevNwDTO&list=PL4F6AB1A2CEB01DC9&index=1 |
| **Calculus I in 20 Minutes** |
| https://www.youtube.com/watch?v=EX_is9LzFSY&list=PL4F6AB1A2CEB01DC9&index=2 |
| **Applications of the Fundamental Theorem of Calculus FRQ YouTube Playlist (54 Videos)** |
| http://www.youtube.com/playlist?list=PL823769CE78C2EE4 |
| **Conceptual Exercises on the Fundamental Theorem of Calculus YouTube Playlist (8 Videos)** |
| http://www.youtube.com/playlist?list=PL20AA72C487C29CE3 |
| **Rectangular and Trapezoidal Approximation FRQ Slideshow with Video Links** |
| **Integration Multiple Choice Slideshow with Solutions** |
| http://www.slideshare.net/jaflint718/ap-calculus-bc-integration-multiple-choice-practice-solutions |
Applications of the Fundamental Theorem of Calculus
FRQ Slideshow with Video Links
http://www.slideshare.net/jaflint718/ap-calculus-ftc-applications-frq
Integration FRQ Slideshow with Video Links
http://www.slideshare.net/jaflint718/ap-integration-frq-problem-set

For suggestions on emphases, classroom activities, labs, projects, and writing assignments see:

- Teaching AP Calculus (McMullin), Ch. 12-15, 17
- Instructor’s Manual Calculus 8th Edition (Anton et al), Ch. 6, 7
- Teacher’s Guide with Answers: Calculus (Finney, et al), Ch. 5
- Learning By Discovery: A Lab Manual for Calculus (Solow), Lab 11-16, 18
- Discovering Calculus with Graphing Calculators (McCarter), Projects 24-31
- Calculus Calculator Labs Workbook (Albert et al), Lab 10, 12, 14, 15
- Calculus Activities: TI-83 Plus, TI-84 Plus Families (Antinone et al), Activity 15-17
- Calculus Problems for a New Century (Fraga), Section V, VI
- Problems for Student Investigation (Jackson et al), Ch. II, III, p. 57-126
- Connecting Mathematics with Science: Experiments for Calculus (Lublinskaya), Ch. 4, 7-8
- A Watched Cup Never Cools (Kamischke), Lab Activity 9, 11, 12-19; Writing Assignment 6-7, 10-11
<table>
<thead>
<tr>
<th>Sample AP-Style unit assessments:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Calculus AP Edition 9th Edition (Larson et al), AP4-1, AP7-1, AP Test Prep Exercises</td>
</tr>
<tr>
<td>• Rogawski’s Calculus for AP Early Transcendentals 2nd Edition (Rogawski et al), Preparing for the AP Exam AP5-1, AP6-1</td>
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<tr>
<th>Problem Solving Extensions and Enrichment:</th>
</tr>
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<tbody>
<tr>
<td>• Single-Variable Calculus: Concepts &amp; Contexts with Vector Functions (Stewart) p. 437, 496, Focus on Problem Solving</td>
</tr>
</tbody>
</table>
## Unit V: Integration and Differential Equations

### Enduring Understandings

Unlike derivatives (which can be found algebraically for any function or expression), antiderivatives can only be found algebraically for a small subset of functions and expressions. Hundreds or even thousands of strategies have been developed over the centuries to find algebraic solutions to different types of differential equations (i.e. to find anti-derivatives). In AP Calculus we learn and emphasize only those with the most broad and far-reaching applications. Beyond that, we learn several techniques to approximate solutions to differential equations. These approximating techniques themselves use different aspects of calculus and make use of modern technology to accomplish what would not have been practically possible in the past.

### Essential Questions

- In mathematics, how much time and effort should be spent learning algebraic techniques when technology exists to perform them?
- When does learning these techniques give students greater insight into the big ideas of mathematics and when does emphasis on technique distract from those ideas?
- At what point might it be useful to use available technology to apply mathematical ideas to solve real-world problems rather than just learning more mathematical skills?

### Knowledge

**Students will know:**

The techniques of integration by substitution can be used to find antiderivatives of expressions involving function composition and trigonometry.

Various trigonometric identities can be used to find antiderivatives of trigonometric functions for which no other analytic solutions exist.

Slope fields are a means of visualizing general and particular solutions to differential equations. These are especially useful when no other analytic solution exists.

Some of the more recent applications of calculus involve exponential growth and decay which can be found in biology, chemistry, environmental science and finance. The technique of separation of variables can be used to find solutions to differential equations involving exponential growth and decay and other expressions.

### Skills

**Students will be able to:**

Use the various techniques of integration by substitution to find antiderivatives of expressions involving function composition and trigonometry.

Use various trigonometric identities to find antiderivatives of trigonometric functions for which no other analytic solutions exist.

Construct slope fields and graph approximate particular solutions to differential equations for which no other analytic solutions exist.

Use the technique of separation of variables to find solutions to differential equations involving exponential growth and decay and other expressions.

### CCSS

<table>
<thead>
<tr>
<th>HSN.RN.1,2</th>
<th>HSN.Q.3</th>
<th>HSN.CN.1</th>
<th>HSA.SSE.1,2,3</th>
<th>HSA.APR.1-4,6,7</th>
<th>HSA.CED.1,2</th>
<th>HSA.REI.1-4,6,7,10</th>
<th>HSF.IF.1-3-5,7-9</th>
<th>HSF.BF.1-3-5</th>
<th>HSF.LE.1-5</th>
<th>HSS.TF.1-4,8,9</th>
<th>HSG.SRT.2,5-8</th>
<th>HSG.C.5</th>
<th>HSG.GMD.1-4</th>
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<th>SMP.1-8</th>
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<table>
<thead>
<tr>
<th>SUGGESTED TIME ALLOTMENT</th>
<th>CONTENT UNIT OF STUDY</th>
<th>SUPPLEMENTAL UNIT RESOURCES</th>
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</thead>
<tbody>
<tr>
<td>7 weeks</td>
<td>Unit V – Integration</td>
<td>For AP Exam Free Response Questions on Integration, see ap13_calc_fraq_coll_index (FRQ Cross-Reference Index) in the AP Calculus Resource Binder</td>
</tr>
<tr>
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<td>o Integration by Substitution</td>
<td>For AP Exam Multiple Choice Questions on Integration, see ap06_calc_mcindx_incl_2008 (Multiple Choice Cross-Reference Index) in the AP Calculus Resource Binder</td>
</tr>
<tr>
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<td>o Trigonometric Integrals</td>
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<td>o Slope Fields</td>
<td>Motion: A Curriculum Module for AP Calculus</td>
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<td>o Exponential Growth and Decay</td>
<td>The Domain of Solutions to Differential Equations</td>
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<td>Don’t Forget the Differential Equations: Finishing 2005 BC4</td>
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<tr>
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<td>AP Calculus: Slope Fields</td>
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<td><a href="https://www.youtube.com/watch?v=dDHhevNwDTo&amp;list=PL4F6AB1A2CEB01DC9&amp;index=1">https://www.youtube.com/watch?v=dDHhevNwDTo&amp;list=PL4F6AB1A2CEB01DC9&amp;index=1</a></td>
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Calculus I in 20 Minutes
https://www.youtube.com/watch?v=EX_is9LzFSY&list=PL4F6AB1A2CEB01DC9&index=2

Integration by Substitution Slideshow with Video Links
http://www.slideshare.net/jaflint718/integration-by-substitution-10670059

Slope Fields and Differential Equations Slideshow with Video Links
http://www.slideshare.net/jaflint718/slope-fields-and-differential-equations

Integration by Substitution You Tube Playlist (9 Videos)
http://www.youtube.com/playlist?list=PLE6B7A374D2F17E06

For suggestions on emphases, classroom activities, labs, projects, and writing assignments see:

- Teaching AP Calculus (McMullin), Ch. 16, 18-20
- Instructor’s Manual Calculus 8th Edition (Anton et al), Ch. 6, 8, 9
- Teacher’s Guide with Answers: Calculus (Finney, et al), Ch. 5, 7, 8
- Learning By Discovery: A Lab Manual for Calculus (Solow), Lab 17, 25
- Discovering Calculus with Graphing Calculators (McCarter), Project 38
- Calculus Calculator Labs Workbook (Albert et al), Lab 9, 11, 13
- Calculus Activities: TI-83 Plus, TI-84 Plus Families (Antinone et al), Activity 13-14
- Calculus Problems for a New Century (Fraga), Section IV
- Problems for Student Investigation (Jackson et al), Ch. II p. 57-92
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<td>Calculus AP Edition 9th Edition (Larson et al), AP5-1, AP6-1, AP7-1 AP Test Prep Exercises</td>
<td>A Watched Cup Never Cools (Kamischke), Lab Activity 13-14, 18; Writing Assignment 6, 10-11</td>
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<td>Rogawski's Calculus for AP Early Transcendentals 2nd Edition (Rogawski et al), Preparing for the AP Exam AP7-1, AP8-1</td>
</tr>
<tr>
<td>Problem Solving Extensions and Enrichment:</td>
<td>Problem Solving Extensions and Enrichment:</td>
</tr>
</tbody>
</table>
### ENDURING UNDERSTANDINGS

| SOLVING A MATHEMATICAL PROBLEM INVOLVES MAKING SENSE OF WHAT IS KNOWN AND APPLYING A THOUGHTFUL PROCESS AND LOGICAL PROCESS, WHICH SOMETIMES REQUIRES PERSEVERANCE, FLEXIBILITY, AND A BIT OF INGENUITY. |
| A WELL-CRAFTED ARGUMENT REQUIRES A THOUGHTFUL AND LOGICAL PROGRESSION OF MATHEMATICALLY SOUND STATEMENTS AND SUPPORTING EVIDENCE. |
| RECOGNIZING A STRUCTURE OR PATTERN CAN BE THE KEY TO SOLVING A PROBLEM OR MAKING SENSE OF A MATHEMATICAL IDEA |

### ESSENTIAL QUESTIONS

| HOW CAN “WHAT IS KNOWN” HELP DETERMINE “WHERE TO BEGIN” OR “WHAT TO DO NEXT” WHEN SOLVING A PROBLEM? |
| HOW CAN MATHEMATICAL REASONING BE SUPPORTED? |
| HOW CAN IDENTIFYING A PATTERN OR STRUCTURE HELP THE SOLUTION PROCESS? |

### KNOWLEDGE

**Students will know:**

The fundamental skills and concepts of differential and integral calculus as described in the above curriculum.

### SKILLS

**Students will be able to:**

Effectively apply the knowledge and skills developed throughout the course to solve a variety of multiple choice and free-response questions based on the above curriculum.

### CCSS

<table>
<thead>
<tr>
<th>SMP 1</th>
<th>SMP 2</th>
<th>SMP 3</th>
<th>SMP 4</th>
<th>SMP 5</th>
<th>SMP 6</th>
<th>SMP 7</th>
<th>SMP 8</th>
</tr>
</thead>
</table>
## Unit VIII – AP Examination Preparation

- **Limits, Continuity, Differentiability**
- **Differentiation**
- **Applications of the Derivative**
- **The Definite Integral**
- **Integration**
- **Calculus and Infinite Series**
- **Calculus of Parametric, Vector, and Polar Functions**

### Supplemental Unit Resources

- For AP Exam Free Response Questions by topic, see ap13_calc_frq_coll_index (FRQ Cross-Reference Index) in the AP Calculus Resource Binder
- For AP Exam Multiple Choice Questions by topic, see ap06_calc_mcindx_incl_2008 (Multiple Choice Cross-Reference Index) in the AP Calculus Resource Binder
- [Calculus Formulas Review](https://www.youtube.com/watch?v=dDHhevNwDTo&list=PL4F6AB1A2CEB01DC9&index=1)
- [Calculus I in 20 Minutes](https://www.youtube.com/watch?v=EX_is9LzFSY&list=PL4F6AB1A2CEB01DC9&index=2)
- [1969 AP Calculus AB Multiple Choice Slideshow with Video Links](http://www.slideshare.net/jaflint718/ap-calculus-ab-1969-mc-questions)
- [On the Role of Sign Charts in AP Calculus Exams](http://apcentral.collegeboard.com/apc/members/courses/teachers_corner/36693.html)
- [AP Calculus: Use of Graphing Calculators](http://apcentral.collegeboard.com/apc/members/courses/teachers_corner/2109.html)
Collection of Free-Response Questions 1999-2013
http://apcentral.collegeboard.com/apc/members/exam/exam_information/8031.html


For Full-Length AP Calculus BC Practice Exams see:

- Be Prepared for the AP Calculus Exam (M. Howell, M. Montgomery)
- Fast Track to a 5: Preparing for the AP Calculus AB and BC Examinations (S. Cade, R. Caldwell, J. Lucia)
- Multiple Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination (D. Lederman)
- Pearson Education AP Test Prep Series: AP Calculus (Barton, Brunsting, Diehl, Hill, Tyler, Wilson)
- Barron’s AP Calculus (S. O. Hockett, D. Bock)
- Cracking the AP Calculus AB & BC Exams, 2013 Edition (College Test Preparation) (D. Kahn)
- Kaplan AP Calculus AB & BC 2013-2014 (Kaplan AP Series) (T. Lefcourt Ruby, J. Sellers, L. Korf, J. Van Horn, M. Munn)
<table>
<thead>
<tr>
<th>ENDURING UNDERSTANDINGS</th>
<th>ESSENTIAL QUESTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculus has had a profound effect on the world over the past four hundred years.</td>
<td>• Is Calculus still relevant in today’s world of science, technology, and engineering, and if so, in what ways? If not, why not?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>KNOWLEDGE</th>
<th>SKILLS</th>
<th>CCSS</th>
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</thead>
</table>
| Students will know:                                | Students will be able to:              | SMP 1  
| At least one specific, real-world application of Calculus. | Demonstrate in writing a thorough understanding of the application of skills and concepts Calculus applies to real-world applications via teacher assigned projects. | SMP 2  
|                                                     |                                      | SMP 3  
|                                                     |                                      | SMP 4  
|                                                     |                                      | SMP 5  
|                                                     |                                      | SMP 6  
|                                                     |                                      | SMP 7  
|                                                     |                                      | SMP 8  
|                                                     |                                      | ELA-Literacy.WHST.11-12.2  
|                                                     |                                      | ELA-Literacy.WHST.11-12.4  
|                                                     |                                      | ELA-Literacy.WHST.11-12.6  
|                                                     |                                      | ELA-Literacy.WHST.11-12.8  
|                                                     |                                      | ELA-Literacy.RST.11-12.3  
|                                                     |                                      | ELA-Literacy.RST.11-12.4  
|                                                     |                                      | ELA-Literacy.RST.11-12.5  
|                                                     |                                      | ELA-Literacy.RST.11-12.10  

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<th>SUGGESTED TIME ALLOTMENT</th>
<th>CONTENT – UNIT OF STUDY</th>
<th>SUPPLEMENTAL UNIT RESOURCES</th>
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</table>
| 4 weeks                  | Unit IX – Extensions and Enrichment  
  o Limits, Continuity, Differentiability  
  o Differentiation  
  o Applications of the Derivative  
  o The Definite Integral  
  o Integration  
  o Calculus and Infinite Series  
  o Calculus of Parametric, Vector, and Polar Functions | Resources for Student Projects:  
  • Connecting Mathematics with Science: Experiments for Calculus (Irina E. Lyublinskaya)  
  • Calculus Problems for a New Century (Robert Fraga, Editor)  
  • Applications of Calculus (Philip Straffin, Editor)  
  • A Watched Cup Never Cools: Lab Activities for Calculus & Precalculus (Ellen Kamischke)  
  • Learning By Discovery: A Lab Manual for Calculus (Anita E. Solow, Editor)  
  • Problems for Student Investigation (M. B. Jackson, J. R. Ramsay, Editors) |
RESOURCES:

Textbook:

Calculus: Graphical, Numerical, Algebraic
Author: R. Finney, F. Demana, B. Waits, D. Kennedy
Copyright 2012 Pearson Education, Inc.

Additional Textbooks:

Rogawski’s Calculus for AP
Author: J. Rogawski, R. Cannon
Copyright 2012 W.H. Freeman and Company

Author: R. Larson, B. Edwards
Copyright 2010 Brooks/Cole, Cengage Learning

Single-Variable Calculus: Concepts & Contexts with Vector Functions
Author: J. Stewart
Copyright 2007 Thomson Brooks/Cole

Calculus: Single-Variable Early Transcendentals
Author: W. Briggs, L. Cochran, B. Gillett
Copyright 2011 Pearson Education, Inc.

Calculus 8th Edition: Early Transcendentals Single Variable
Author: H. Anton, I. Bivens, S. Davis
Copyright 2005 Anton Textbooks, Inc.
Calculus: Concepts and Applications  
Author: Paul A. Foerster  
Copyright 2005 Key Curriculum Press

Calculus from Graphical, Numerical, and Symbolic Points of View: Single Variable, 2nd Edition  
Author: A. Ostebee, P. Zorn  
ISBN: 0-03-025681-X  
Copyright 2002 Thomson Learning, Inc.

Calculus Single Variable, 4th Edition  
ISBN: 0-471-48481-4  
Copyright 2005 John Wiley & Sons

Cross-Curricular / Project Based Learning / Exploration Resources:

Connecting Mathematics with Science: Experiments for Calculus  
Author: Irina E. Lyublinskaya  
Copyright 2003 Key Curriculum Press

Calculus Problems for a New Century  
Author: Robert Fraga, Editor  
Copyright 1993 Mathematical Association of America

Applications of Calculus  
Author: Philip Straffin, Editor  
ISBN: 0-88385-085-0  
Copyright 1993 Mathematical Association of America

A Watched Cup Never Cools: Lab Activities for Calculus & Precalculus  
Author: Ellen Kamischke  
Copyright 1999 Key Curriculum Press
Learning By Discovery: A Lab Manual for Calculus
Author: Anita E. Solow, Editor
ISBN: 0-88385-083-4
Copyright 1993 Mathematical Association of America

Problems for Student Investigation
Author: M. B. Jackson, J. R. Ramsay, Editors
ISBN: 0-88385-086-9
Copyright 1993 Mathematical Association of America

Graphing Calculator Supplements:

Discovering Calculus with Graphing Calculators
Author: Joan H. McCarter
Copyright 1992 John Wiley & Sons, Inc.

Calculus Activities: TI-84 Plus, TI-83 Plus Families
Author: L. Antinone, T. Dick, K. Fitzpatrick, M. Grasse, M. Howell
Copyright 2004 Texas Instruments Incorporated

Calculus Calculator Labs Workbook
Author: B Albert, P. Hillis
ISBN: 0-9727055-S-L
Copyright 2005 Skylight Publishing

AP Calculus with the TI-83 Graphing Calculator
Author: G. Best, S. Fischbeck
Copyright 1998 Venture Publishing

Instructor Resources:

Teaching AP Calculus
Author: Lin McMullin
Copyright 2005 D&S Marketing Systems, Inc.
Instructor’s Manual Calculus Early Transcendentals 8th Edition
Authors: H. Anton, I. Bivens, S. Davis
Copyright 2005 John Wiley & Sons

Teacher’s Guide with Answers: Calculus
Author: R. L. Finney, F. D. Demana, B. K. Waits, D. Kennedy
ISBN: 0-201-32446-6
Copyright 1999 Addison Wesley Longman, Inc.

AP Exam Preparation Books:

Be Prepared for the AP Calculus Exam
Author: M. Howell, M. Montgomery
ISBN: 0-9727055-5-4
Copyright 2005 Skylight Publishing

Fast Track to a 5: Preparing for the AP Calculus AB and BC Examinations
Author: S. Cade, R. Caldwell, J. Lucia
ISBN: 0-618-14944-9
Copyright 2006 Houghton Mifflin Company

Multiple Choice & Free-Response Questions in Preparation for the AP Calculus (BC) Examination
Author: D. Lederman
Copyright 2005 D&S Marketing Systems, Inc.

Pearson Education AP Test Prep Series: AP Calculus
Author: Barton, Brunsting, Diehl, Hill, Tyler, Wilson
Copyright 2012 Pearson Education, Inc.

Barron’s AP Calculus
Author: S. O. Hockett, D. Bock
Copyright 2010 Barron’s Educational Series, Inc.
Cracking the AP Calculus AB & BC Exams, 2013 Edition (College Test Preparation)
Author: D. Kahn
Copyright 2012 The Princeton Review, Inc.

Kaplan AP Calculus AB & BC 2013-2014 (Kaplan AP Series)
Author: T. Lefcourt Ruby, J. Sellers, L. Korf, J. Van Horn, M. Munn
Copyright 2012 Kaplan Publishing, Inc.

Technology:

- Blackboard Course Management Software
- Graphing Calculator
- Winplot.exe
- TI-SmartView Emulator Software
- iPad apps for TI-Nspire, Wolfram Calculus
- Presentation software such as Powerpoint

Web addresses:

College Board AP Calculus BC Home Page
ASSESSMENT:

- Quiz
- Test
- Problem Sets
- Individual Projects
- Group Projects
- Homework
- Online Resources
Opportunities exist for interdisciplinary units with courses such as Physics, Biology, Chemistry, Astronomy and other science electives.
It is assumed that the student has successfully completed Algebra I, Geometry, Algebra II, and Precalculus or the equivalent.