Randolph Township Schools
Randolph High School

Discrete Mathematics Curriculum

"The essence of mathematics is not to make simple things complicated, but to make complicated things simple." ~ S. Gudder

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Curriculum Developed
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Randolph Township Schools
Department of Science, Technology, Engineering, and Math

Discrete Mathematics

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Randolph Township Schools

Mission Statement

It is the mission of the Randolph Township Schools to help prepare all our students for further education, productive work, responsible leadership, and personal fulfillment. Toward that end, we will provide students with educational experiences that enable them to acquire the knowledge and develop the thinking and problem–solving skills necessary for a lifelong process of learning. We will guide all students in discovering, valuing, and developing their unique talents in order to realize their potential.

Randolph Township Schools
Affirmative Action Statement

Equality and Equity in Curriculum

The Randolph Township School district ensures that the district's curriculum and instruction are aligned to the State's Core Curriculum Content Standards. The curriculum addresses the elimination of discrimination and the achievement gap, as identified by underperforming school-level AYP reports for State assessment. The curriculum provides equity in instruction, educational programs and provides all students the opportunity to interact positively with others regardless of race, creed, color, national origin, ancestry, age, marital status, affectional or sexual orientation, gender, religion, disability or socioeconomic status.

N.J.A.C. 6A:7-1.7(b): Section 504, Rehabilitation Act of 1973; N.J.S.A. 10:5; Title IX, Education Amendments of 1972

RANDOLPH TOWNSHIP BOARD OF EDUCATION
EDUCATIONAL GOALS
VALUES IN EDUCATION

The statements represent the beliefs and values regarding our educational system. Education is the key to self-actualization, which is realized through achievement and self-respect. We believe our entire system must not only represent these values, but also demonstrate them in all that we do as a school system.

We believe:

- The needs of the child come first
- Mutual respect and trust are the cornerstones of a learning community
- The learning community consists of students, educators, parents, administrators, educational support personnel, the community and Board of Education members
- A successful learning community communicates honestly and openly in a non-threatening environment
- Members of our learning community have different needs at different times. There is openness to the challenge of meeting those needs in professional and supportive ways
- Assessment of professionals (i.e., educators, administrators and educational support personnel) is a dynamic process that requires review and revision based on evolving research, practices and experiences
- Development of desired capabilities comes in stages and is achieved through hard work, reflection and ongoing growth

Randolph Township Schools
Department of Science, Technology, Engineering, and Math

Introduction

Randolph Township Schools is committed to excellence. We believe that all children are entitled to an education that will equip them to become productive citizens of the 21st century. We believe that an education grounded in the fundamental principles of science, technology, engineering, and math (STEM) will provide students with the skills and content necessary to become future leaders and lifelong learners.

A sound STEM education is grounded in the principles of inquiry, rigor, and relevance. Students will be actively engaged in learning as they use real-world STEM skills to construct knowledge. They will have ample opportunities to manipulate materials and solve problems in ways that are developmentally appropriate to their age. They will work in an environment that encourages them to take risks, think critically, build models, observe patterns, and recognize anomalies in those patterns. Students will be encouraged to ask questions, not just the “how” and the “what” of observed phenomena, but also the “why”. They will develop the ability, confidence, and motivation to succeed academically and personally.

STEM literacy requires understandings and habits of mind that enable students to make sense of how the our world works. As described in Project 2061’s *Benchmarks in Science Literacy, The Standards for Technological Literacy,* and *Professional Standards for Teaching Mathematics,* literacy in these subject areas enables people to think critically and independently. Scientifically and technologically literate citizens deal sensibly with problems that involve mathematics, evidence, patterns, logical arguments, uncertainty, and problem-solving.

Discrete Mathematics

Introduction

This curriculum is based on the belief that mastery in learning takes place over an extended period of time. Students will learn to value mathematics; recognize reoccurring themes across mathematical domains; strengthen mathematical proficiency through problem solving, inquiry, and discovery; learn to communicate and reason mathematically; and create mathematical representations through the use of technology. Students learn analytical techniques as a basis for development and use of mathematical models to reflect real life applications and to foster a life-long learning and appreciation for mathematics.

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Curriculum Pacing Chart
Discrete Mathematics

SUGGESTED TIME ALLOTMENT	UNIT NUMBER	CONTENT - UNIT OF STUDY
3 weeks	I	Voting Methods
3 weeks	II	Weighted Voting Systems
3 weeks	III	Mathematics of Apportionment
3 weeks	IV	Fair Division
3 weeks	V	Graph Theory
3 weeks	VI	Income

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Discrete Mathematics
UNIT I: Voting Methods

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
Methods for counting votes lie at the heart of the democratic process.	• Why should we vote?
The winner of a vote can depend on the method chosen to count the votes.	• What is the best method for conducting an election when there are more than two candidates?
A method for determining election results that is democratic and always fair is a mathematical impossibility.	• Does each vote really count? • How do we know that no perfectly fair voting method exists?

KNOWLEDGE	SKILLS	CCSS
Students will know: The difference between a winner of a vote by plurality and a winner with majority. The benefits of using a preference table as opposed to counting each ballot individually. Using the methods of borda count, plurality with elimination, and pairwise comparison, each take into consideration a voter's rank of each choice. In any vote involving more than two choices, there is no voting method that will satisfy all of the four fairness criteria.	**Students will be able to:** Distinguish between winning with a majority of the votes and winning with a plurality of votes. Create a preference table by putting together all voters preference ballots. Determine the winner of a vote using each method: • Plurality • Borda Count • Plurality with Elimination • Pairwise Comparison Explain why each voting method passes or fails the fairness criteria: • Majority Criterion • Condorcet's Criterion • Independence of Irrelevant Alternatives Criterion • Monotonicity Criterion	9-12.Q.1 9-12.Q.2 9-12.REI.1 9-12.IF.4 9-12.MD.7 SMP.1-8 ELA.RST.11-12.3 ELA.RST.11-12.4 ELA.RST.11-12.7 ELA.RST.11-12.8 ELA.RST.11-12.9 ELA.WHST.11-12.2 ELA.WHST.11-12.4 ELA.WHST.11-12.7 ELA.WHST.11-12.9

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Unit I - Curriculum Pacing Chart
Discrete Mathematics

SUGGESTED TIME ALLOTMENT	CONTENT – UNIT OF STUDY	SUPPLEMENTAL UNIT RESOURCES
3 weeks	**Unit I – Voting Methods** ○ **Preference Tables** ○ **Plurality method** ○ **Borda Count method** ○ **Plurality with Elimination method** ○ **Pairwise Comparison method** ○ **Majority Criterion** ○ **Condorcet's Criterion** ○ **Independence of Irrelevant Alternatives Criterion** ○ **Monotonicity Criterion**	**Textbook I : Chapter 1** **Textbook II : Chapter 10.1 and 10.2** **Textbook III : Section 14.1 and 14.2** **Individual Election** **Fast Food Vote example** **School Wide Election**

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Discrete Mathematics
UNIT II: Weighted Voting Systems

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
In any society, no matter how democratic, some individuals and groups have more power than others.	• When is the principle of one person, one vote not just?
Diversity is the inherent reason the concept of power exists.	• What is the purpose of a weighted voting system? • How is a voter's power measured?

KNOWLEDGE	SKILLS	CCSS
Students will know:	**Students will be able to:**	
The numeric representation of a weighted voting system.	Determine the quota of a vote and the number of votes for each voter.	9-12.Q.2 9-12.SSE.1 9-12.REI.1 SMP.1-7
There can be more than one winning coalition.	Put each voter into a coalition and determine which coalitions are winning coalitions.	ELA.RST.11-12.3 ELA.RST.11-12.4
	Determine the critical voters of each winning coalition.	ELA.RST.11-12.7 ELA.RST.11-12.8
The number of votes a voter has does not represent how powerful they are as an individual.	Calculate the Banzhaf power index of each voter and explain its meaning.	ELA.RST.11-12.9 ELA.WHST.11-12.2 ELA.WHST.11-12.4 ELA.WHST.11-12.7 ELA.WHST.11-12.9

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Unit II - Curriculum Pacing Chart
Discrete Mathematics

SUGGESTED TIME ALLOTMENT	CONTENT – UNIT OF STUDY	SUPPLEMENTAL UNIT RESOURCES
3 weeks	Unit II – Weighted Voting Systems ○ Quota and Weights ○ Winning Coalitions ○ Critical Voter ○ Banzhaf Power Index	Textbook I : Chapter 2 Textbook II : Chapter 10.3 United Nations Security Council example Jury example Small Town Politics example Electoral College example Krook, Cheatum & Associates Law Firm example

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Discrete Mathematics
UNIT III: Mathematics of Apportionment

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
The best apportionment method depends on which outcome you prefer.	• Is there an apportionment method that yields a fair distribution? Why or why not?
Each apportionment method is flawed.	• What paradoxes can occur when applying the methods of apportionment?

KNOWLEDGE	SKILLS	CCSS
Students will know: The apportionment of items can change depending on the method for apportioning that is used. An apportionment method that does not violate the quota rule and does not produce any paradoxes is a mathematical impossibility.	**Students will be able to:** Explain apportionment using five different methods: • Hamilton • Jefferson • Adams • Webster • Huntington Hill Determine and discuss the flaws in each method: • Violation of quota rule • Alabama paradox • Population paradox • New-states paradox	9-12.Q.3 9-12.REI.1 9-12.IF.4 9-12.IC.6 9-12.MD.7 SMP.1-7 ELA.RST.11-12.3 ELA.RST.11-12.4 ELA.RST.11-12.7 ELA.RST.11-12.8 ELA.RST.11-12.9 ELA.WHST.11-12.2 ELA.WHST.11-12.4 ELA.WHST.11-12.7 ELA.WHST.11-12.9

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Unit III - Curriculum Pacing Chart
Discrete Mathematics

SUGGESTED TIME ALLOTMENT	CONTENT – UNIT OF STUDY	SUPPLEMENTAL UNIT RESOURCES
3 weeks	**Unit III – Mathematics of Apportionment** ○ History of the House of Representatives ○ Hamilton method ○ Standard divisor ○ Modified divisor ○ Jefferson method ○ Adams method ○ Webster method ○ Huntington-Hill method ○ Quota rule ○ Alabama paradox ○ Population paradox ○ New-states paradox ○ Balinski and Young's Impossibility Theorem	**Textbook I : Chapter 4** **Textbook II : Chapter 9.1 to 9.4** **Textbook III : Section 14.3 and 14.4** **Rapid Transit Service example** **Nurse Shifts example** **Police Precincts example** **The First Apportionment of the House of Representatives example** **The 2000 Presidential Election example**

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Discrete Mathematics
UNIT IV: Fair Division

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
Mathematics allows people to determine how to share in a reasonable and fair way.	• Is there a way something that must be shared by a set of competing parties be divided among them in a way that ensures each party receives a fair share? How?
It is important to understand under what circumstances each fair division method can and cannot be used.	• What is a fair share? • How can an indivisible object (or set of objects) be fairly divided?

KNOWLEDGE	SKILLS	CCSS
Students will know: The Divider-Chooser method is always the method of choice for a continuous fair division problem involving two players. The methods of the Lone-Divider, the Lone-Chooser, and the Last-Diminisher are all good choices for a continuous fair division problem involving three or more players. The methods of Markers and Sealed Bids are the best choices for a discrete fair division problem involving two or more players.	**Students will be able to:** Distribute a divisible item between two players using the Divider-Chooser method. Distribute a divisible item among three or more players using the following methods: • Lone-Divider • Lone-Chooser • Last-Diminisher Divide an indivisible item(s) among two or more players using the following methods: • Markers • Sealed Bids	9-12.Q.3 9-12.CED.1 9-12.REI.1 9-12.IC.6 9-12.MD.7 SMP.1-8 ELA.RST.11-12.3 ELA.RST.11-12.4 ELA.RST.11-12.7 ELA.RST.11-12.8 ELA.RST.11-12.9

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Unit IV - Curriculum Pacing Chart
Discrete Mathematics

SUGGESTED TIME ALLOTMENT	CONTENT – UNIT OF STUDY	SUPPLEMENTAL UNIT RESOURCES
3 weeks	**Unit IV – Fair Division** ○ Fair shares ○ Divider-Chooser method ○ Lone Divider method ○ Lone Chooser method ○ Last Diminisher method ○ Method of Markers ○ Method of Sealed Bids	**Textbook I : Chapter 3** **Cake Cutting examples** **Newly Discovered Island example** **Grandma's Will example** **Halloween example**

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Discrete Mathematics
UNIT V: Graph Theory

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS
Relationships can be modeled with graphs in order to solve a variety of real world problems.	• Why are graphs used to represent real world relationships and situations? • How do we determine the most efficient solution to a problem where a graphical model is used?

KNOWLEDGE	SKILLS	CCSS
Students will know: Euler's Theorem can be used on graph models to solve real world problems. A Hamilton circuit can be used when determining the most efficient solutions. Directed graphs model relationships that go in only one direction.	**Students will be able to:** List all Euler paths and circuits of a graph modeling a real world scenario and explain their significance. Apply Fleury's Algorithm to construct Euler circuits. List all Hamilton paths of a graph and their weights. Apply the following algorithms on Hamilton paths to a graph: • Brute Force • Nearest Neighbor • Best Edge Find paths in a directed graph.	9-12.Q.2 9-12.REI.1 9-12MG.1 9-12.MG.3 9-12.MD.7 SMP.1-8 ELA.RST.11-12.3 ELA.RST.11-12.4 ELA.RST.11-12.7 ELA.RST.11-12.8 ELA.RST.11-12.9 ELA.WHST.11-12.2 ELA.WHST.11-12.4 ELA.WHST.11-12.7 ELA.WHST.11-12.9

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Unit V - Curriculum Pacing Chart
Discrete Mathematics

SUGGESTED TIME ALLOTMENT	CONTENT – UNIT OF STUDY	SUPPLEMENTAL UNIT RESOURCES
3 weeks	**Unit V – Graph Theory** ○ **Euler Paths and Circuits** ○ **Hamilton Paths and Circuits** ○ **Directed Paths**	**Textbook I : Chapter 5 and 6** **Textbook II : Chapter 3** **Textbook III : Section 15.1, 15.2, and 15.3** **Traveling Salesman Problem** **Konigsberg Bridge Problem** **Four Color Problem for the US and South America** **Scheduling of Committees example** **Modeling Influence example** **Scheduling Projects**

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Discrete Mathematics
UNIT VI: Income

<table>
<tr><th>ENDURING UNDERSTANDINGS</th><th>ESSENTIAL QUESTIONS</th></tr>
<tr><td>Salary is not the same as net income.</td><td>How do you choose the best career to make you happy? How do you choose the best career to make a great deal of money? How do you choose a career that accomplishes both?</td></tr>
</table>

<table>
<tr><th>KNOWLEDGE</th><th>SKILLS</th><th>CCSS</th></tr>
<tr><td>Students will know:

Aptitude assessments help to identify fields in which one will likely excel.

Net income results after involuntary payments are deducted from gross salary.

It is necessary to plan for Savings as well all other living expenses.</td><td>Students will be able to:

Take an aptitude assessment to identify career fields to consider.

Research fields identified in aptitude assessment and identify:
Educational required
Skills required
Working conditions
Activities in a typical day
Locations of employment
Career paths available
Salary (starting, median, top decile)
Share these finds with the entire class.

Calculate net income, weekly, bi-weekly, and monthly, after the following required withholdings:
Federal taxes
State taxes
Social Security
Medicare
Research living expenses and estimate their dollar cost:
Automobile and insurance
Rent or mortgage
Cell phone
Food</td><td>9-12.Q.3
9-12.SSE.1
SMP.3-7

ELA.RST.11-12.4
ELA.RST.11-12.7
ELA.RST.11-12.8
ELA.RST.11-12.9
ELA.WHST.11-12.2
ELA.WHST.11-12.4
ELA.WHST.11-12.7
ELA.WHST.11-12.9</td></tr>
</table>

	• Utilities • Savings	

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Unit VI - Curriculum Pacing Chart
Discrete Mathematics

SUGGESTED TIME ALLOTMENT	CONTENT – UNIT OF STUDY	SUPPLEMENTAL UNIT RESOURCES
3 weeks	**Unit VI – Income** ○ **Research a career** ○ **Find net income** ○ **Determine weekly, biweekly, and monthly pay** ○ **Research costs of living**	**Laptops** **Federal Income Tax chart** **State Income Tax chart**

RANDOLPH TOWNSHIP SCHOOL DISTRICT
Discrete Mathematics

APPENDIX A

RESOURCES:

Textbook I :
Excursions in Modern Mathematics
Author: Tannenbaum, Peter
ISBN13: 0-13-231913-6
Copyright 2007 Pearson Education, Inc.

Textbook II :
Mathematics All Around
Author: Pirnot, Thomas L.
ISBN:)-13-195997-2
Copyright 2007 Pearson Education, Inc.

Textbook III :
Thinking Mathematically
Author: Blitzer, Robert
ISBN: 0-13-158839-7
Copyright 2008 Pearson Education, Inc.

Technology:
- Spreadsheet software such as Excel
- Word processor software such as Word
- Presentation software such as Power point
- Graphing calculator
- Laptops

APPENDIX B

ASSESSMENT:

- Quiz
- Test
- Individual Projects
- Group Projects
- Homework

APPENDIX C

Opportunities exist for interdisciplinary units with courses such as American History, Sociology, Political Science, Biology and Personal Finance.

APPENDIX D

It is assumed that the student has successfully completed Algebra I, Geometry and Algebra II, or the equivalent.